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AN APPLICATION OF THE PRODUCTION FUNCTION TO THE STUDY OF THE MANUFACTURING INDUSTRY IN MEXICO (1939 - 1955) AND THE U.S.A. (1899 - 1915)

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SUMMARY

The specific purpose of this investigation is to study quantitatively the nature of manufacturing industry in Mexico and the U.S.A. for the respective periods 1939 - 1955 and 1899 - 1915.

The approach to such an objective is to isolate primary intervening factors and observe their functional relationship as related to a measure of activity in manufacturing industry. The choice of such factors or inputs and the output measure follows the lines of input-output and production functional analyses.

The above methodology is much used in econometrics and the resulting models find applications in the economic and industrial engineering fields depending on the particular objectives.

A mathematical model is developed for each country, including five exogenous variables for assessing their relative importance in generating manufacturing output. Economic and statistical adjustments are performed on the data for each variable. The functional relationship is hypothesized followed by reasonable corroboration. Multiple regression techniques as well as correlation are used to develop the final forms of the models and the analysis of such models. The significance of the factors is also studied.

The production elasticities of inputs, the nature of

returns to each and to the function as a whole, and the quantitative and economic applications of regression coefficients are topics covered. A brief mathematical treatment delving into the marginal characteristics of the power functions (models) obtained is presented. Throughout the discussion attention is drawn to the formulation of economic policy and the planning criteria that arise from such analysis.

The results are useful in proportion to the degree of sophistication in the methodology applied. The resulting models are essentially predictor models.

CHAPTER I

INTRODUCTION

This study is an experiment in extending a relatively unexploited tool of analytical economics, namely the production function.

The technical relation between various inputs and outputs is a common engineering problem. The optimum utilization of resources for the maximization of output or profit is one of the primary objectives of industrial engineering.

The law describing such phenomena is in a broad sense that of the production function. The quantitative determination of factors and relationships involved in the mechanism of production can be applied either to a firm or to a broader economic sector. Both lend themselves to investigations involving the production function.

The isolation of factors alone affords grounds for theoretical considerations. However, their quantification provides a more solid foundation for development of decision tools for practical measurement and application. We approach then an analogy between the operations research approach in industrial engineering and econometric methods in the field of analytical economics.

These two disciplines provide quantitative techniques for assessing the status of operations in relation to past performance and in formulating criteria for planning growth and development.

It is the purpose of this study to analyze the nature of the manufacturing industry in Mexico and the United States for the periods 1939 - 1955 and 1899 - 1915, respectively. These periods are considered similar in the course of economic development of the two nations. The selection and quantification of the main inputs or factors in manufacturing will then allow us to formulate mathematical models describing the laws of production for the periods considered for each country.

In broad terms, a basis is provided for judging the relative importance of intervening inputs, their specific quantitative contributions to the growth of manufacturing, and the optimum use or "mix" of resources. The manufacturing policy implications of such models are many.

A detailed discussion of the above problems follows.

CHAPTER II

THE PRODUCTION FUNCTION: BACKGROUND AND SOME THEORETICAL CONCEPTS

The pioneering work of Professors Douglas and Cobb[1] in the third and fourth decades of this century marks the practical approach in attempting to derive laws of production. They sought to obtain a functional relationship between physical output of production in manufacturing, and two inputs, namely, quantity of labor and capital investment.

Let

P = physical output of production per unit time

C = capital investment per unit time

L = number of wage earners per unit time

then:

$$P = f\,(C,\ L) \tag{1}$$

Under the assumption that the sum of the exponents equals one, they obtained:

$$P = f\,(C,\ L) = b\ C^{1-k}\ L^{k} \tag{2}$$

With the use of index numbers and by the method of least squares, the constants were fitted and the resulting equation was:

$$P = 1.01\ C^{0.25}\ L^{0.75} \tag{3}$$

for the period 1899 - 1922 in the United States.

The results computed from this formula agree very closely with the actual data. However, the deviations become greater if projections past 1922 are attempted. This limits our predictor, P, to short range considerations.

Douglas did further work for the State of Massachusetts and the State of New South Wales, Australia, to test the validity of his original results and in answer to criticisms of a diverse nature. To many critics, the inter-correlation of factors, among themselves and with time, simply invalidated the results obtained. However, all of Douglas' studies produced concordant results, and checks were performed to substantiate findings. Checks such as formulation of functions for one specific year for comparison with the time-series approach were marked by success.

The functions were subjected to mathematical treatment for derivation of marginal productivities and production elasticities for each factor. Such questions as the following were investigated:

(1) What is the degree of response in production to changes in the quantities of labor and capital?

(2) What is the relative influence upon output of changes in the quantity of labor as compared with changes in the quantity of capital?

(3) Does the actual course of the distribution of the output of industry between capital and labor approximate what we might expect from our analysis of production?

In time, the economic policy implications of the production function has awakened much interest. The quantitative measurement of variables, tempered by theoretical economic concepts and common sense, might well point to decision models for practical applications.

The derivation of production functions and estimation of structural parameters in the equations may become a basis for assessing the real influence of the resources (or inputs) considered. We can visualize the following specific areas for investigations of this type:

(1) Quantitative measurement of past performance in order to provide benchmarks for determining how efficiently resources are being used in manufacturing.

(2) Formulation of criteria for planning and delineation of objectives in analyzing:

    (a) Optimum quantity of resources to use, and

    (b) Evaluation of importance of particular inputs in manufacturing.

There are diverse approaches to the adaptation of a particular production function for a particular situation. They may briefly be described as follows:[2]

(1) <u>The production function with a single input.</u>

Let

Y = output per unit time

X = resource input per unit time

a, b, c = constant coefficients

then the following equations are most convenient:

$$Y = aX^b \tag{4}$$

$$Y = aX \pm bX^2 \pm cX^3 \tag{5}$$

$$Y = aX \pm b\sqrt{X} \tag{6}$$

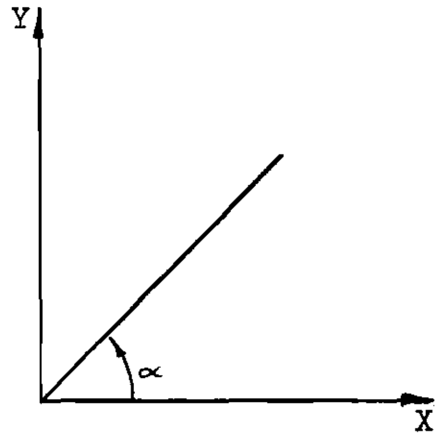
and the general form is

$$Y = f\,(X) \tag{7}$$

Equation (4) is a logarithmic function of the Cobb-Douglas type. It has the following properties:

(a) If b is equal to one, we say that the elasticity of production is one, and this means that we have constant returns accruing to any increases in input. This case is illustrated in Figure 1.

(b) If b is less than one, we have the case of decreasing returns, i.e., output increases by a smaller percentage than input and variable costs per unit increase. This case is illustrated in Figure 2.



Note:

$a = \tan \alpha$

Figure 1. Production Function with Constant Elasticity of One

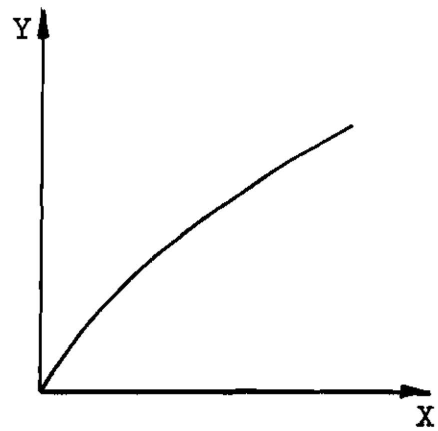


Figure 2. Production Function with Elasticity Less Than One

(c) If b is greater than one, we have the case of increasing returns, i.e., output increases by a greater percentage than input and consequently variable costs per unit decline. This case is illustrated in Figure 3.

Equation (5) with the linear, squared, and cubic terms, has greater flexibility than the Cobb-Douglas function. It lends itself nicely to describe a fairly complex relationship between input and output as illustrated by Figure 4.

Equation (6) with linear and square root terms is an alternative which may be used instead of Equation (5) for analyzing a production function with varying magnitudes of elasticity.

(2) <u>The production function with two inputs</u>.

Let

Y = output per unit time

X_1 = quantity of one input per unit time

X_2 = quantity of another input per unit time

a,b,c,d,e = constant coefficients

then we may consider the following equations:

$$Y = aX_1^b \, X_2^c \tag{8}$$

$$Y = aX_1 \pm bX_1^2 \pm cX_2 \pm dX_2^2 \pm eX_1X_2 \tag{9}$$

$$Y = aX_1 \pm b\sqrt{X_1} \pm cX_2 \pm d\sqrt{X_2} \pm e\sqrt{X_1X_2} \tag{10}$$

and the general form is

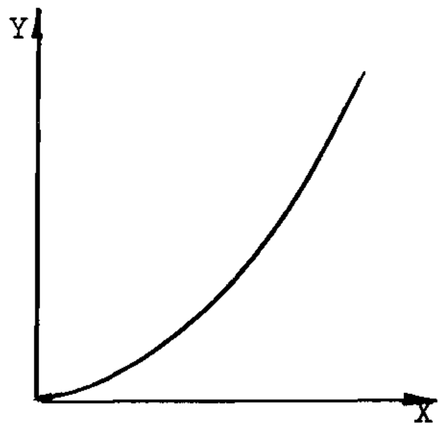
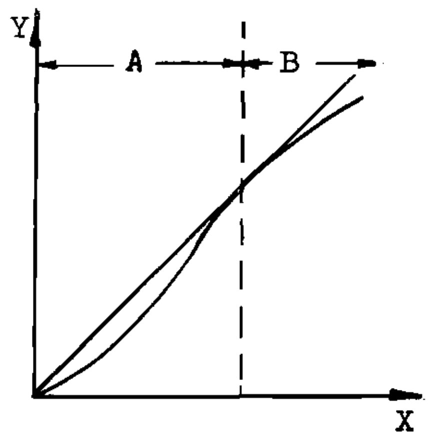


Figure 3. Production Function with Elasticity Greater Than One



Note:

A: Region where elasticity of production is greater than one.

B: Region where elasticity of production is less than one.

Figure 4. Production Function with Elasticity of Varying Magnitude

$$Y = f\ (X_1, X_2) \tag{11}$$

The properties of a production function with two variable inputs can be visualized by means of an iso-product contour map. This map consists of a family of curves, each contour corresponding to a different level of output. The contour map for a two input production function must satisfy the following conditions:[3]

(a) If either input is held constant while the other is increased (decreased), output will increase (decrease).

(b) If output is held constant, a decrease (increase) in one output will require an increase (decrease) in the other input.

(c) If the output is held constant, the marginal rate at which X_1 substitutes for X_2 increases as X_1 increases.

The contour map which satisfies these conditions is illustrated in Figure 5.

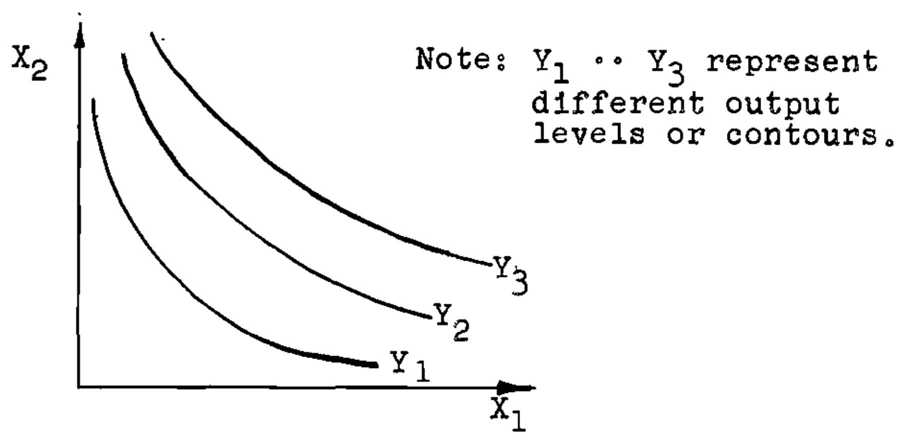


Figure 5. Iso-Product Map for Two-Input Production Function

A further aspect must be explained, e.g., the nature of factor-factor (input-input) relationships for specific output levels. These exhibit the following characteristics:

Case (a): The factors (inputs) are imperfect substitutes, i.e., both inputs must be employed in some proportion for a given output. Furthermore, they can be substituted without limit. The following diagram illustrates this condition:

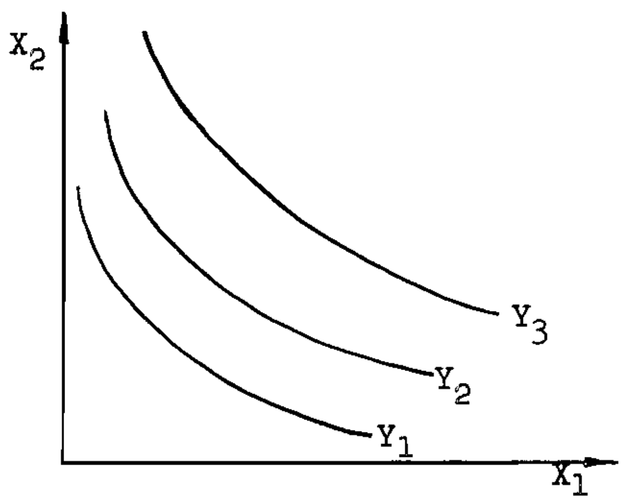


Figure 6. Iso-Product Map for Imperfect Substitutes

Case (b): The factors are perfect substitutes, i.e., one factor may completely substitute another in producing a given output. The following diagram illustrates this condition:

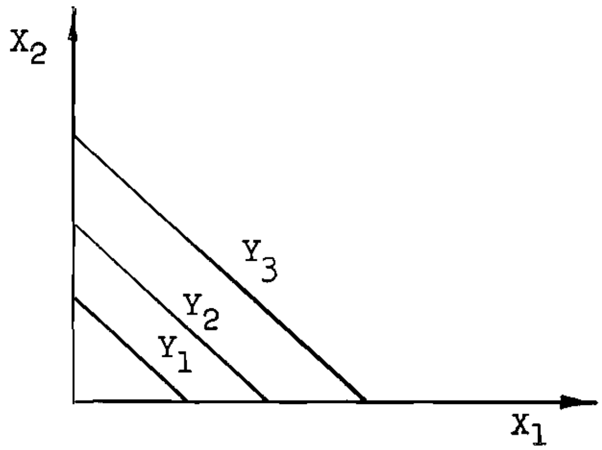
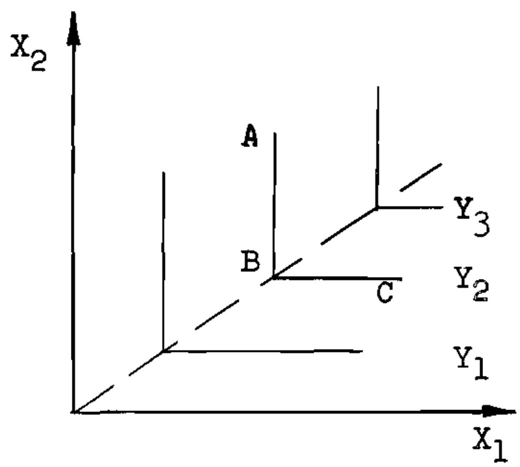


Figure 7. Iso-Product Map for Perfect Substitutes

<u>Case (c)</u>: The factors are limitational inputs, i.e., if a specified quantity of product is to be produced then a certain unique quantity of each input must be used. Excesses of any one input contribute nothing to production, while shortages of any one input limit the output that can be produced. The following diagram will illustrate this condition:



Note: The segment $\overline{AB}$ represents the excesses of input X_2 at output Y_2. The segment $\overline{BC}$ represents the excesses of input X_1 at output Y_2.

Figure 8. Iso-Product Map for Limitational Inputs

Case (d): The factors can be substituted for each other but not without limits. The following diagram illustrates this condition:

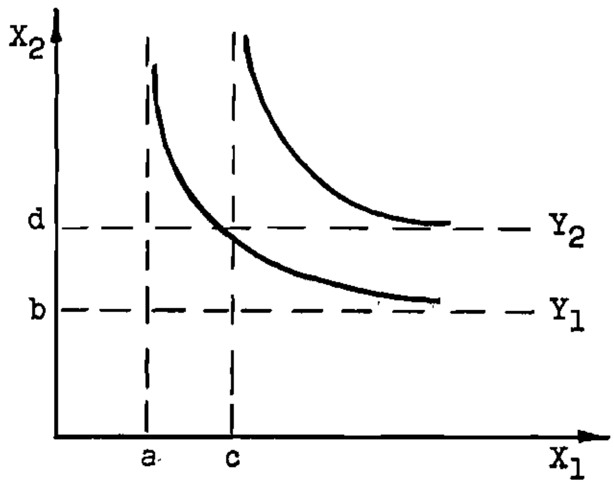


Figure 9. Iso-Product Map for Limited Substitution Inputs

The diagram tells us that it is not possible to produce output Y_1 with less than "a" units of input X_1, nor less than "b" units of input X_2.

(3) The production function with multiple inputs.

If the number of inputs is three, four, five, and so on, the most commonly used equation is the Cobb-Douglas production function. Thus, for example, for four inputs we have:

$$Y = aX_1^b X_2^c X_3^d X_4^e \tag{12}$$

In the above equation, a is a constant while the exponents b,c,d, and e indicate the nature of returns for its corresponding resource. If "b" is less than one, an increase in X_1 will result in diminishing returns to the particular resource. For "b" equal to one, we have constant returns for increases in X_1; and for "b" greater than one, we have increasing returns for additions of resource X_1.

The nature of returns for the entire equation, i.e., for all the factors combined, is obtained by adding the exponents (b+c+d+e). The same conditions will hold for this sum as mentioned above for the individual exponents (elasticities).

A power function along the lines of the Douglas-Cobb type may for purpose of illustration be:[4]

$$Y = 8.0\ F^{0.7944} \tag{13}$$

We are using only one factor-input (F) for the output (Y). In the logarithmic form the equation would be:

$$\text{Log } Y = 0.90309 + 0.7944 \text{ Log } F \tag{14}$$

An immediate indication of returns is given by the exponent or elasticity of production .7944.

$$\frac{\partial(\text{Log } Y)}{\partial(\text{Log } F)} = +0.7944 \tag{15}$$

Since it is less than one, decreasing returns are indicated for this hypothetical example.

Questions related to the optimum amount of output and efficient use of resources have as their foundation the study of "marginal conditions." Such conditions imply the addition of each input for the production of a certain output until the additional input fails to pay for its corresponding additional cost. Again, a simple power function such as

$$Y = aX^{\alpha} Z^{\beta} \tag{16}$$

may be used for marginal analysis.

The marginal productivities of the individual inputs are

$$\frac{\partial Y}{\partial X} = a\alpha Z^{\beta} X^{\alpha - 1} \tag{17}$$

$$\frac{\partial Y}{\partial Z} = a\beta X^{\alpha} Z^{\beta - 1} \tag{18}$$

Inferences from production functions help make decision rules specifying the use of this or that resource and whether more or less of each should be used. The criteria will be marginal product and factor-price relations that will be mentioned later.*

It is not possible to consider all of the theoretical concepts related with power production functions within the scope of this study. An attempt has been made to reveal some of the findings in the known research with a brevity that it is hoped may clarify some of the ensuing methodology.

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*<u>Infra</u>., p. 58.

CHAPTER III

THE MANUFACTURING SETTING IN MEXICO (1939-1955) AND IN THE U.S.A. (1899-1915)

To study the manufacturing "plant" in both countries, a similar period had to be chosen. The criteria for selection were: (1) availability of data, and (2) analogous position in the rural-urban population balance.

The periods 1939 - 1955 and 1899 - 1915 were selected for Mexico and the U.S.A., respectively. In general we were able to obtain data for these years; and in those cases where only scattered years were quantified, the curves were plotted and the values interpolated to complete the sets of data. The sources for Mexico differed widely and therefore those which seemed most reliable were consulted.

The rural-urban population proportions are shown in the following table:

Table 1. Rural-Urban Population Proportions in Mexico and the U.S.A.

		Rural Pop.		Urban Pop.		
	Year	Number	%	Number	%	Total
MEX	1940	12,757,411	64.9	6,896,589	35.1	19,654,000
	1950	14,806,623	57.4	10,984,594	42.6	25,791,017

Table 1. (Continued)

	Year	Rural Pop. Number	%	Urban Pop. Number	%	Total
U S A	1900	45,834,654	60.3	30,159,921	39.7	75,994,575
	1910	49,973,334	54.3	41,998,932	45.7	91,972,266

Sources: J. O. Ochoa, Poblacion, Primera Edicion, Mexico, D. F., Fondo de Cultura Economica, 1955, p. 37.

Historical Statistics of the United States, U. S. Department of Commerce, Series B 145-159, p. 29.

The movement of population from rural to urban areas indicates a similar pattern of growth for both countries since we reasonably assume industry and manufacturing to be concentrated in urban centers.

Mexican manufacturing industry is considered to include the following sub-categories: (1) cotton textiles, (2) woolen, (3) rayon, (4) clothing, (5) flour milling, (6) beer, (7) canning and processing foods, (8) vegetable oils, (9) sugar, (10) iron and steel, (11) cement, (12) glass, (13) shoes, (14) soap, (15) tobacco, (16) matches, (17) rubber, (18) paper, and (19) alcohol, and various other industries which fall under the category of "Transformation Industries" in Mexican statistics. In every case, data were secured attempting to cover similarly classified industries for both countries.

The whole of the manufacturing industry, as in the case of its smaller counterpart--the manufacturing firm--revolves about economic as well as institutional factors. Data that might reflect such complex factors have been collected. Those that appear to be more relevant are tabulated below. The other data will be found in the Appendix in Table 16 through Table 24.

The setting of manufacturing is clarified further if we observe the following phenomena:

(1) Investment in manufacturing industry as a per cent of total national investment.

(2) Contribution by manufacturing to gross national product.

(3) Employment in manufacturing as compared to other sectors.

(4) Overall index of physical volume of production in manufacturing.

(5) Productivity index based on physical output and manufacturing employment indices.

The percent investment in manufacturing industry in Mexico for 1939 - 1950 was 22 per cent and approximately the same for the U.S.A. for 1899 - 1915.

The contributions to gross national product for the same periods were 17 and 20 per cent for Mexico and the U.S.A. respectively.

Table 2 gives an interesting picture of the employment in manufacturing relative to the gainfully employed population.

Table 2. Manufacturing and Agriculture: Distribution of Gainfully Employed Population (in thousands)

	Year	Employed in Manufacturing		Employed in Agriculture		Gainfully Employed Population
		Number	%	Number	%	
MEX	1940	630	12	3,626	68	5,352
	1950	973	12	4,824	58	8,346
USA	1900	6,090	21	9,552	33	29,025
	1909	8,446	22	11,599	31	37,454

Sources: United Nations Yearbook, 1957, Table 6, p. 54.
United Nations Yearbook, 1948, Table 7, p. 64.
Historical Statistics of the United States, U. S. Department of Commerce, Series D 62-76, p. 65.

The discrepancy between rural-urban population percentages and those describing the actual employment in agriculture and manufacturing were due to other sectors in the economy which absorb the difference in gainfully employed. Most noteworthy is the sector of commerce. The latter occupies a large percentage of those gainfully employed in Mexico which might otherwise be participating in manufacturing industry. Other sectors that deduct from the possible number of workers that might be associated with manufacturing are mining and construction. However, Table 2 does point out that equality of circumstances as far as real manufacturing potential is concerned does not exist for the corresponding periods studied. We would have to retrace our steps to conditions in the U.S.A. in the 1860's. This has been brought out clearly by M. German Parra, a noted Mexican economist.[5] There is a counter-balancing factor, however, that would reduce the difference in states of development and that is the level of technology. Techniques and methods that were not available at the beginning of the century are available today to evolve an accelerating growth of industry. As a final comment, the tendency for Mexico to follow the line of transformation from an agricultural to an industrial economic structure is indicated by the decrease in employment in agriculture from 1940 to 1950. All other evidence clearly supports this transition.

Again an analogy may be drawn between small scale operations and the aggregate manufacturing "plant." In determining alternative methods of production and analyzing the general condition of operations of an enterprise, we might look into:

(1) Output per worker

(2) Output per unit of capital invested

(3) Amount of labor per unit of output

(4) Amount of capital input per unit of output

(5) Ratio of capital per worker

From physical indices of production we have calculated the coefficients of production for manufacturing industry in Mexico and the U.S.A., as shown in Table 3.

Let

C = Accumulated capital additions to fixed capital (machinery and equipment)

P = Physical volume of production

L = Wage earners in manufacturing industry

All indices are calculated with 1939 = 100 and 1899 = 100 as base years.

The slightly higher productivity for Mexico (P/L), or estimate of the output per worker, is probably due to the advantage of a much improved technology over that which existed in the first decade of the century.

Of the obvious points brought out by Table 3, we note the capital-intensive nature of industrial development in Mexico. The ratios C/P and C/L give such an indication. However, the P/C ratio might indicate excess capacity for under-utilized equipment and/or inefficient use of installed facilities. The P/L ratio should be proportionately higher if full advantage were being taken of current (1950) technology in the process, managerial, and administrative areas. In the United States, given the slow increase in productivity (P/L), the proportionality of the various factors is more

Table 3. Coefficients of Production

Mexico (1939 - 1950)						U.S.A. (1899 - 1910)					
Year	P/C	L/P	C/P	C/L	P/L	Year	P/C	L/P	C/P	C/L	P/L
1939	1.00	1.00	1.00	1.00	1.00	1899	1.00	1.00	1.00	1.00	1.00
1940	.56	1.01	1.79	1.78	.99	1900	.94	1.04	1.06	1.02	.96
1941	.42	.95	2.35	2.50	1.05	1901	.98	.98	1.02	1.04	1.02
1942	.40	.92	2.47	2.70	1.08	1902	1.00	.97	1.00	1.03	1.03
1943	.34	.95	2.94	3.12	1.05	1903	.94	.99	1.06	1.06	1.01
1944	.28	.93	3.51	3.84	1.07	1904	.88	.95	1.13	1.19	1.05
1945	.23	.89	4.34	5.00	1.12	1905	.96	.87	1.04	1.19	1.15
1946	.17	.91	5.74	6.25	1.09	1906	.93	.88	1.07	1.22	1.14
1947	.12	.92	8.10	9.09	1.08	1907	.86	.91	1.16	1.28	1.10
1948	.10	.92	9.48	10.00	1.08	1908	.68	.96	1.47	1.53	1.04
1949	.10	.90	10.21	11.11	1.11	1909	.78	.90	1.28	1.41	1.11
1950	.10	.84	10.07	12.50	1.19	1910	.76	.90	1.31	1.44	1.11

Sources: Comision Mixta, El Desarrollo Economico de Mexico, Nacional Financiera S.A., Fondo de Cultura Economica, 1953, p. 231.

Douglas, P. H., Theory of Wages, 1st Ed., New York, The MacMillan Company, 1934, Table 6, p. 121.

balanced as is to be expected. We pointed out previously that there do exist basic dissimilarities in the period studied.

Mention may be made of further differences, some of which come under the classification of institutional factors. They are: (1) saving habits, (2) educational and training facilities, (3) size of the market, (4) transportation, (5) credit and capital availability, (6) entrepreneurship, (7) resource availability, (8) taxation, (9) industrial research and exchange of information, and (10) other sociological factors.

CHAPTER IV

CONSTRUCTION OF THE MODELS

Having chosen the periods, we proceed with the development of a mathematical model in order to quantify the production relationships. The conceptual framework of the manufacturing unit is to be translated into broader terms, because insofar as input and output are concerned, the mechanism is essentially the same. In the realm of the plant we deal with inputs such as:

(1) Capital (fixed, working)

(2) Materials (raw, semi-finished)

(3) Labor

(4) Management (organizational techniques, application and development of existing technology, etc.)

(5) Power

We may visualize an input-output system with a simple iconic model.

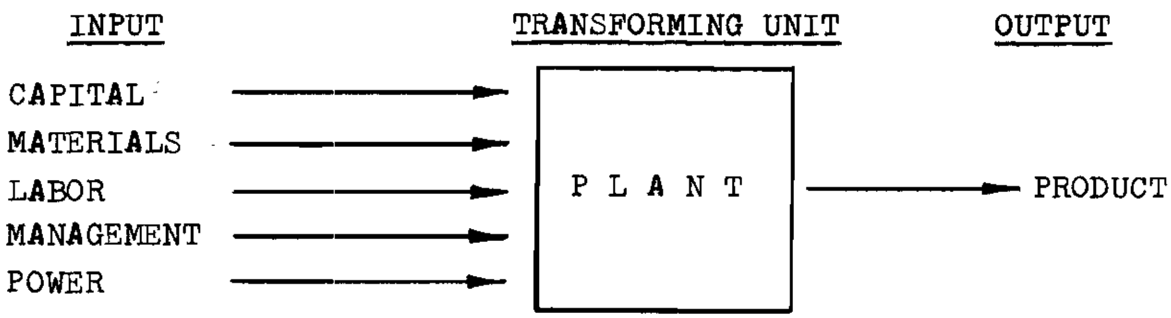


Figure 10. Iconic Model of Production

A transforming unit, i.e. the plant, is fed production factors (inputs) for the purpose of creating an output or product. This is a rudimentary concept and is the basis of our analysis.

In the analysis of manufacturing industry as a macro system, our input components are considered in terms of quantifiable factors in the aggregate. It is well to note that institutional factors are always exercising their influence, but their very nature makes them unquantifiable. Such factors are management and the state or development of technology. We will comment on these later.*

The fundamental factors chosen are then:

(1) Total capital investment

(2) Wages and salaries

(3) Raw material consumption

(4) Power consumption

The endogenous variable we wish to investigate is our product or, in this case, manufacturing output. The factors are not all-inclusive, but represent the essential inputs in manufacturing industry. For an exploratory investigation in the aggregate they are readily measurable. If a single firm were to be analyzed, a more minute breakdown would seem advisable. Furthermore, with a regression model relating inputs and output, it is recommended that the number of variables be reduced to a practical minimum to avoid loss of accuracy in the predictive or structural model.

---

*<u>Infra</u>., p. 27.

At this point we observe that a classical function of the following type may be applied to associate the component parts of our system:

$$\text{Output} = f\ (\text{inputs}) \qquad (19)$$

$$\text{Manufacturing output} = f\ (X_1, X_2, \cdots X_n) \qquad (20)$$

where n = number of inputs

The measurement of each input factor should be oriented toward satisfying the requirements of our larger producing unit--the whole of manufacturing industry. The units of each variable were chosen in the currency of the particular country involved (pesos and dollars). For both models (Mexico and the U.S.A.), data were obtained for each factor by interpolating fitted curves to isolated figures available over the respective periods studied. The obvious nature of the growth and development tendencies within both nations simplified the collection of data by this method.

Total capital investment figures include all fixed asset investment at current prices with the exception of land. We consider in this item that capital which aids directly in the production of goods, and this becomes our criterion for measuring this factor. The value of wages and salaries, raw material, and power consumed by manufacturing were obtained by methods similarly employed for capital investment. The more reliable sources were chosen at the writer's discretion and rarely were the data available for successive years for the periods investigated. Enough observations were available, however, to determine curve characteristics in each case.

We have to contend with two long-run disturbances that would affect our input-output function. They are: (1) time, the dynamic factor, and (2) the changing value of money.

We include time as a factor absorbing the advances in management and technology and other factors not considered explicitly in our analysis. Time is not a causal factor in itself. The effect of the value of money was compensated for by deflating all data in both models by the purchasing power indices with 1939 = 100 and 1899 = 100 as base years.

The aggregate study, as implied by the comments made to this point, is facilitated by the use of index numbers. All data have been transformed to the indices with 1939 and 1899 as base years of one-hundred. This applies to dependent and independent variables. It was found that percentage changes are more meaningful than natural numbers in production analysis.

The next step was to plot each of the output-input relationships to ascertain the product-factor function relationships.

The various time series for Model I - Mexico were highly irregular. Reference to the plots in Figures 11 - 14 will bear this out. Analysis of the data was handled by the

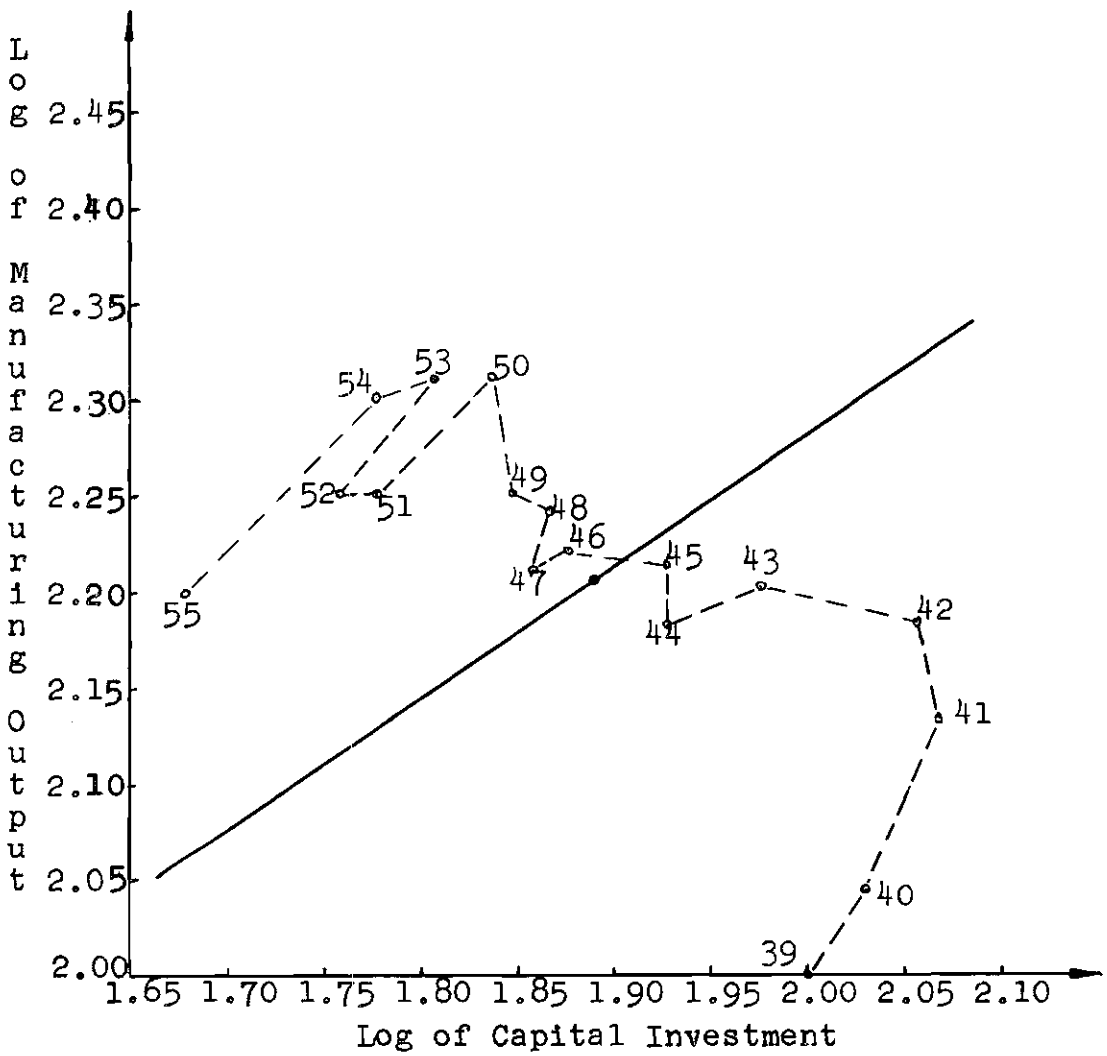


Figure 11. Relation of Manufacturing Output and Capital Investment: Mexico

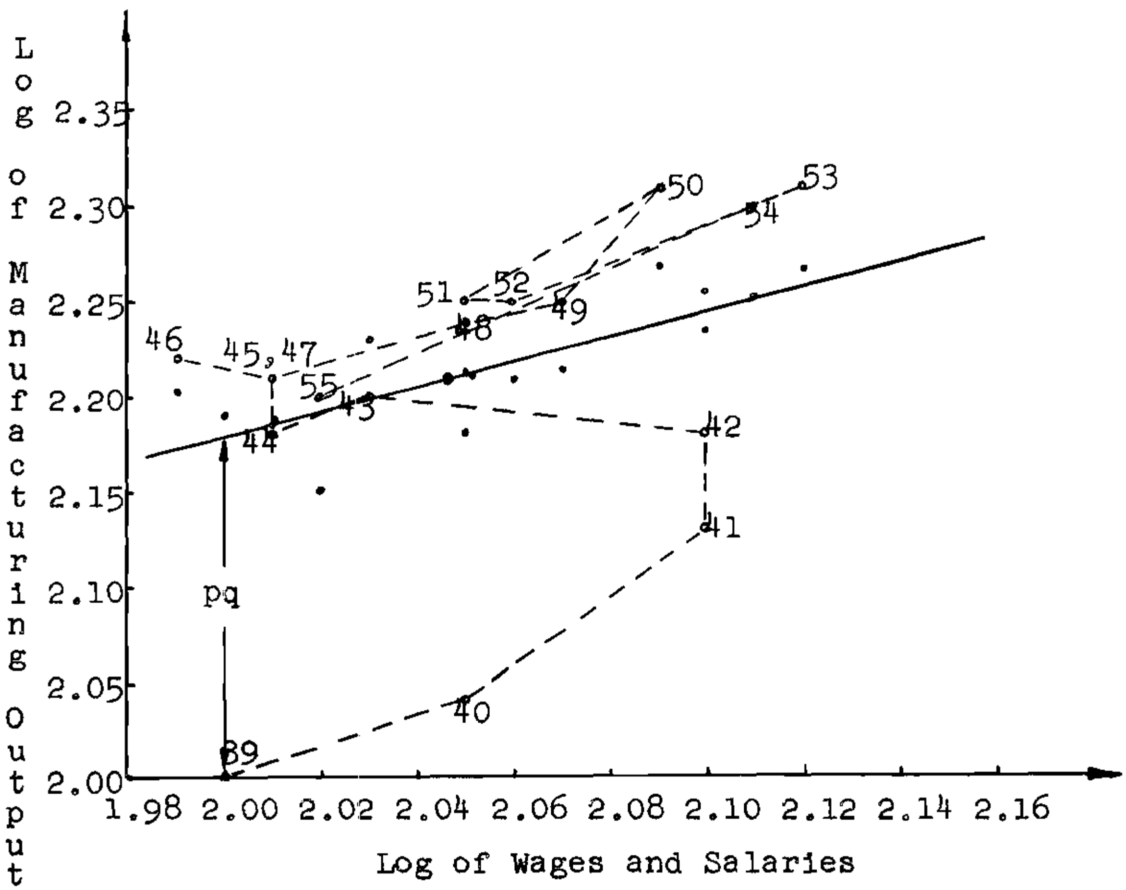


Figure 12. Relation of Manufacturing Output and Wages and Salaries: Mexico

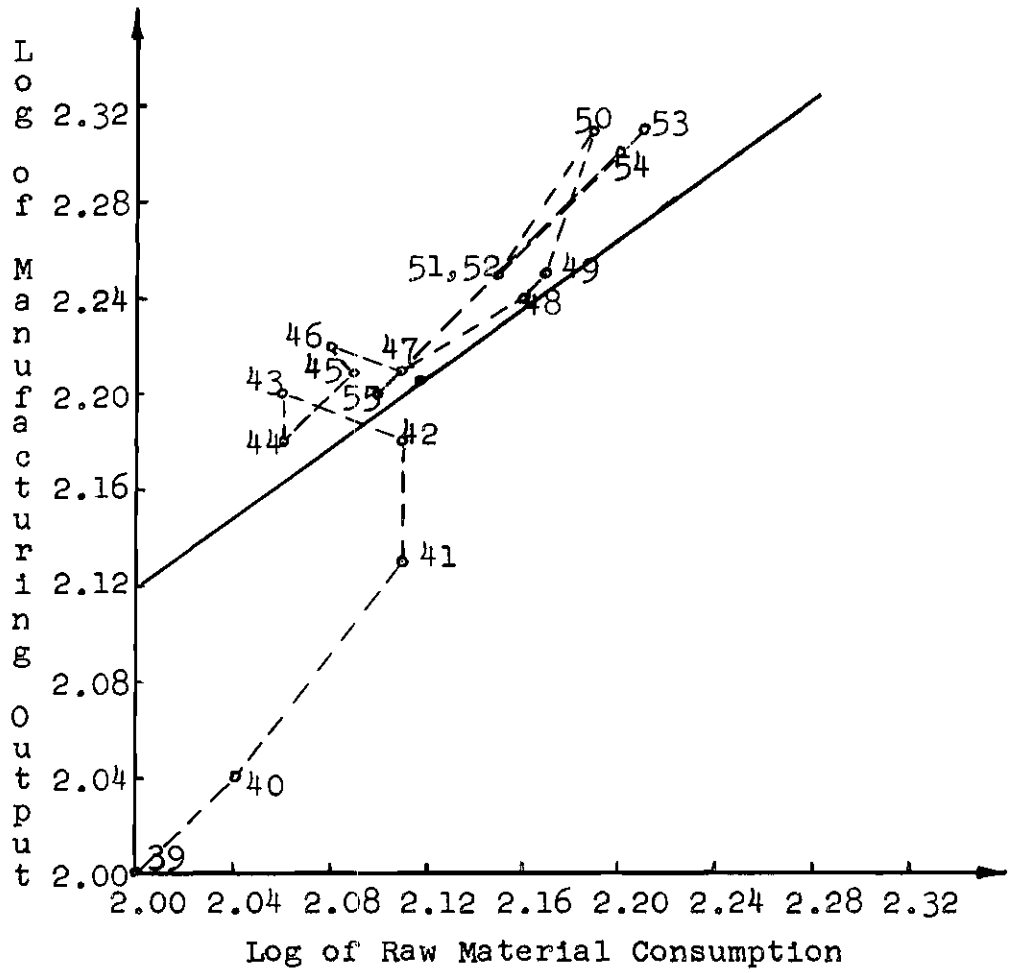


Figure 13. Relation of Manufacturing Output and Raw Material Consumption: Mexico

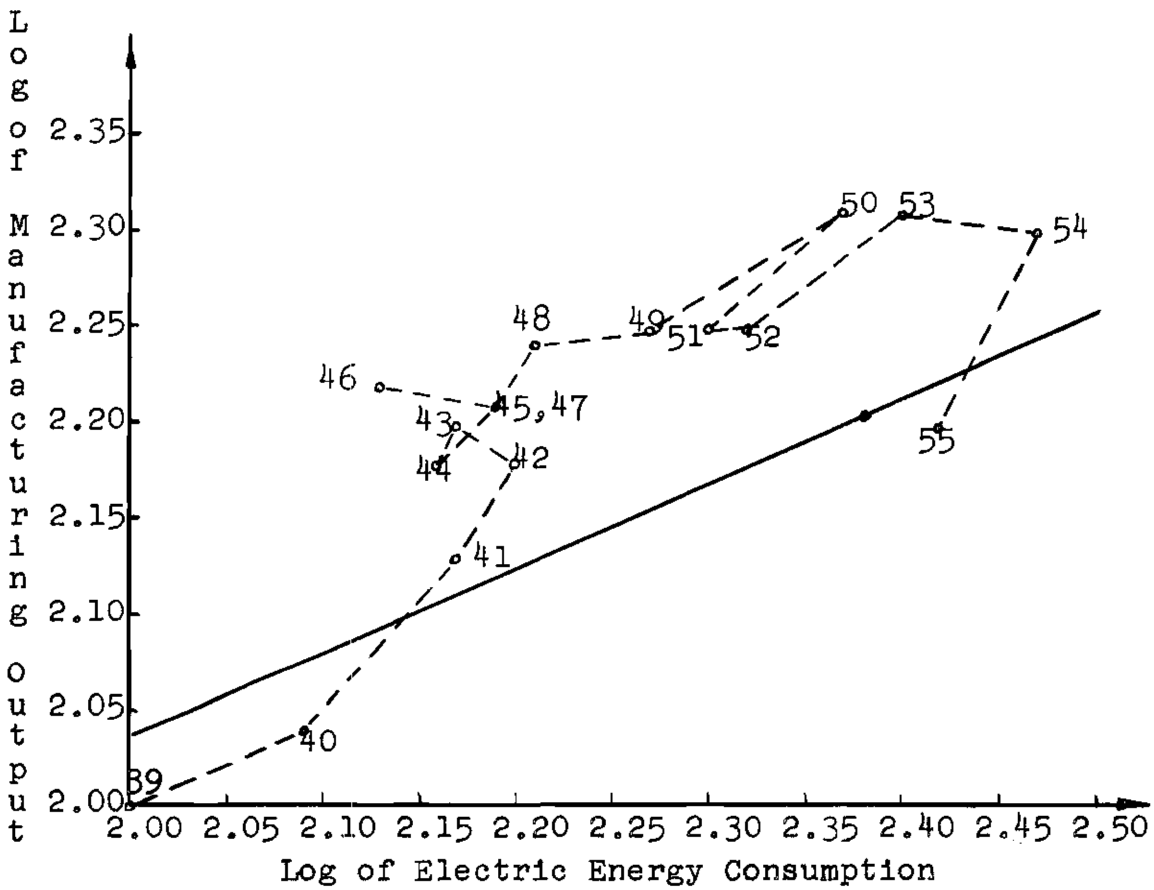


Figure 14. Relation of Manufacturing Output and Electric Energy Consumption: Mexico

Graphic Scaffolding Method.[6] The method entails briefly the following steps:

(1) Assume a reasonable curve to describe the product-factor plot.

(2) The eye-fitted curve should pass through the mean of the two series.

(3) Each observation is measured for its deviation from the hypothesized relationship.

(4) These deviations are plotted from a zero base line as shown in Figure 15.

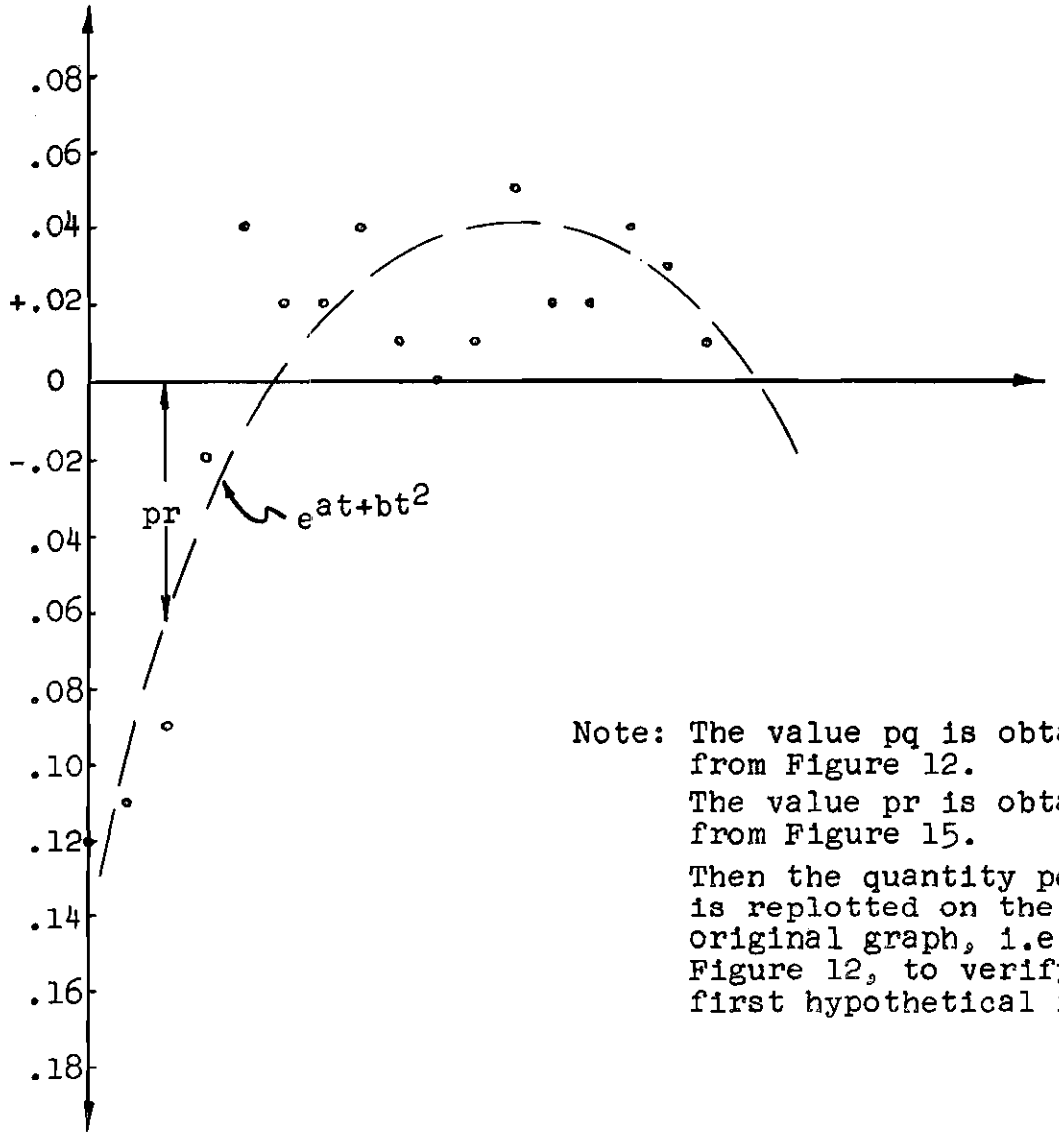
(5) A new set of observations is obtained for which an estimated curve is fitted by eye to satisfy the new plot.

(6) The resulting curve gives us the characteristics of the time element in the data, i.e., linear or parabolic.

(7) The deviations of observations from the fitted curve on this new plot are replotted on the plot of original data to obtain corrected observations (after the exclusion of time).

(8) The modified observations according to step (6) are scrutinized for adequacy of fit of the hypothesized function in the original data.

We hypothesized a linear fit in the logarithmic transformation of each product-factor plot for the Mexican data. The new plots of the deviations were found to be parabolic,



Note: The value pq is obtained from Figure 12.

The value pr is obtained from Figure 15.

Then the quantity pq-pr is replotted on the original graph, i.e., Figure 12, to verify first hypothetical fit.

Figure 15. Graphic Scaffolding Method for Figure 12

and therefore time was considered to enter as a second degree term according to the method of Schultz.[7] The corrected observations were found satisfactory after eliminating the time element and we therefore accepted each product-factor association as linear in the logarithmic transformation.

The data for the U.S.A. were immediately linear in the logarithmic transformation with the exception of the product-factor plot for manufacturing output-power consumption as shown in Figures 16 - 19. The nature of the latter relationship was found to be parabolic in the third degree. For the U.S.A., time as a factor was again investigated by the method described above. There arose an interesting problem at this point. The scatter of the deviation plots did not indicate a specific function for time. The time series could not be devoid of a dynamic factor and another procedure based on Tintner[8] was employed. Each variable was plotted against time in the logarithmic transformation with the original index data. In every case we observed a parabolic relationship as shown in Figure 20. We concluded that time should enter in just such a form, i.e., parabolic in our regression function. Our curiosity is aroused by the comparison to be obtained by using these two approaches in analyzing the time factor. It was decided that the final models would be our measure of effectiveness for judging the Schultz and Tintner methods.

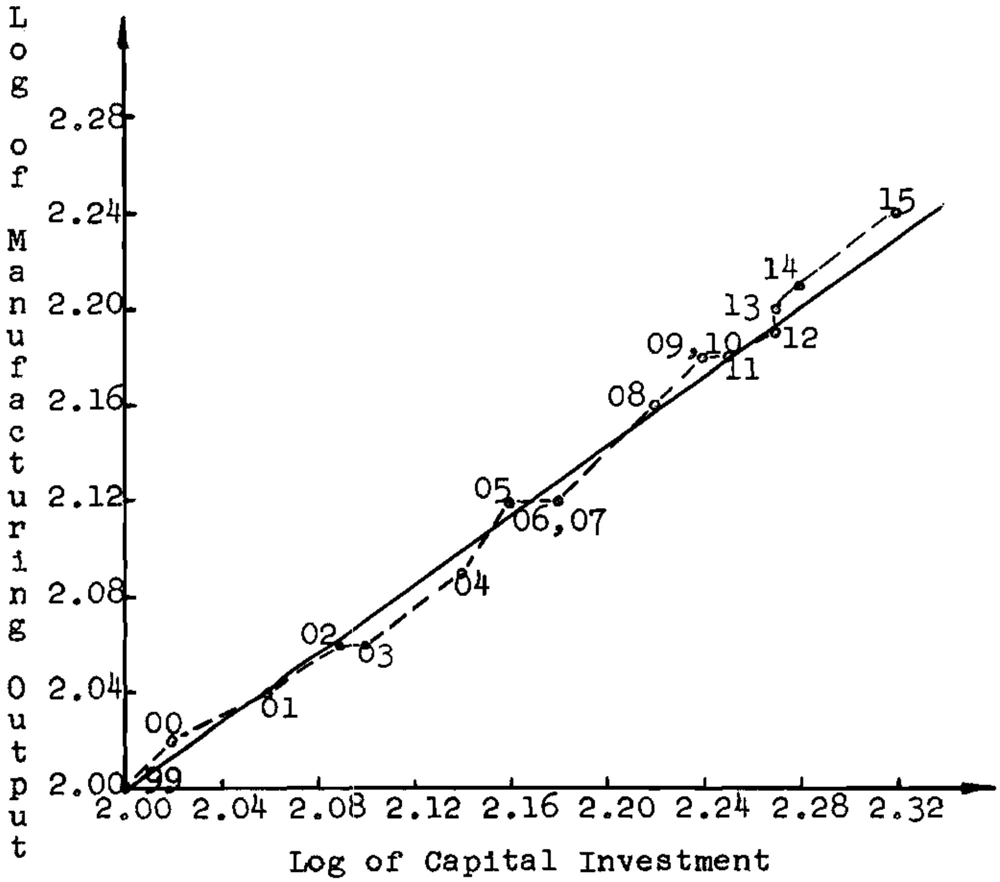


Figure 16. Relation of Manufacturing Output and Capital Investment: U.S.A.

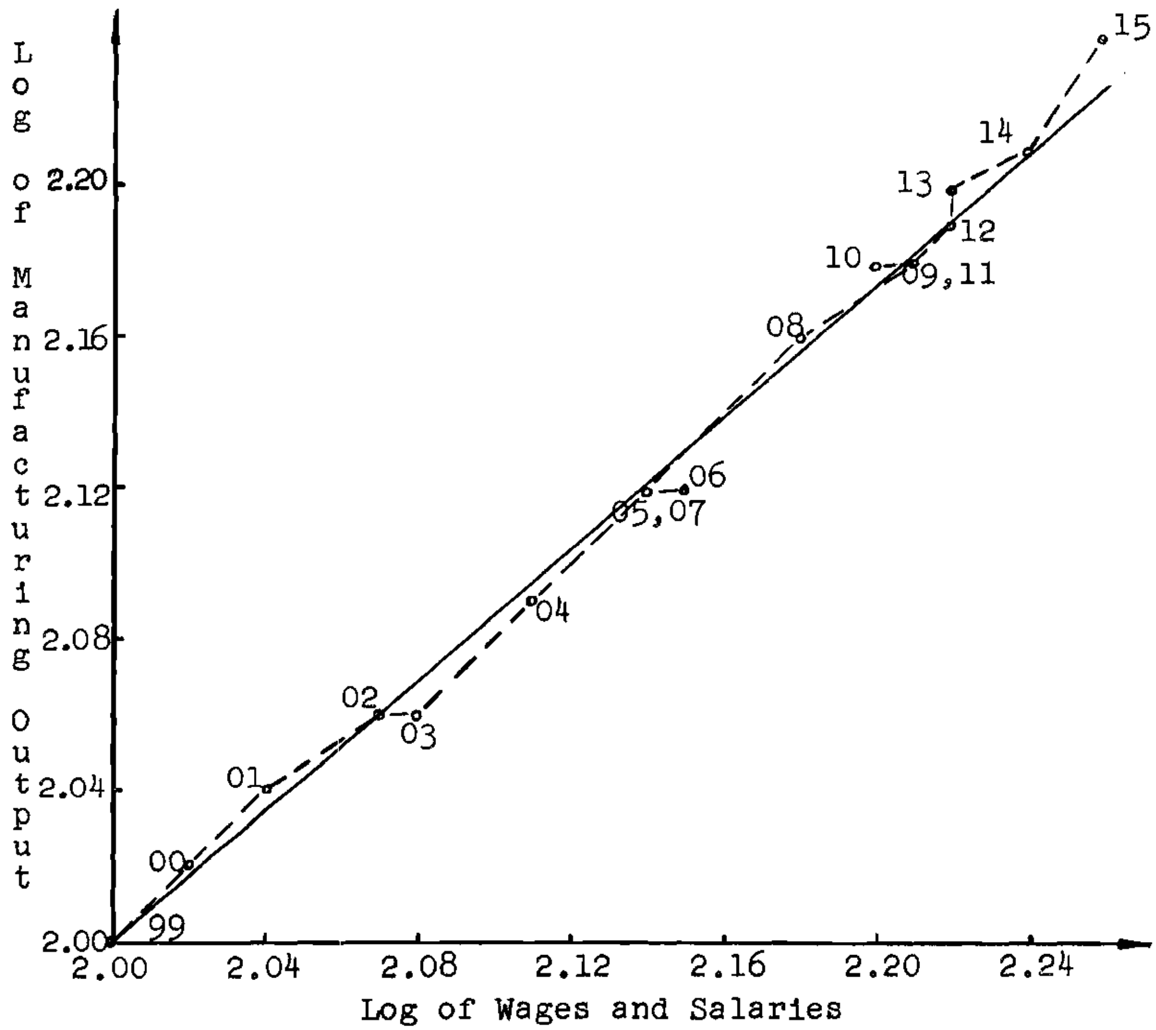


Figure 17. Relation of Manufacturing Output and Wages and Salaries: U.S.A.

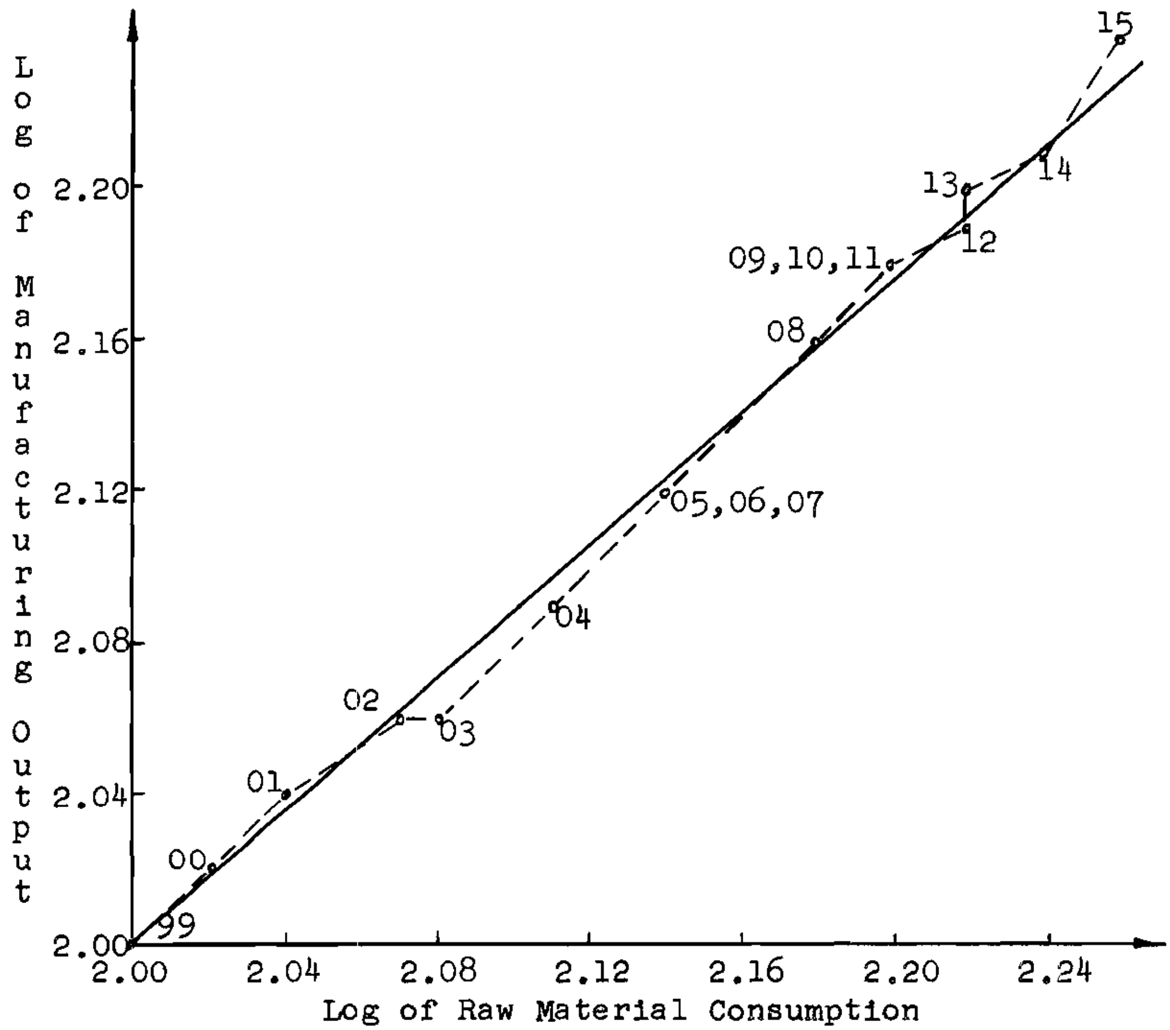


Figure 18. Relation of Manufacturing Output
and Raw Material Consumption: U.S.A.

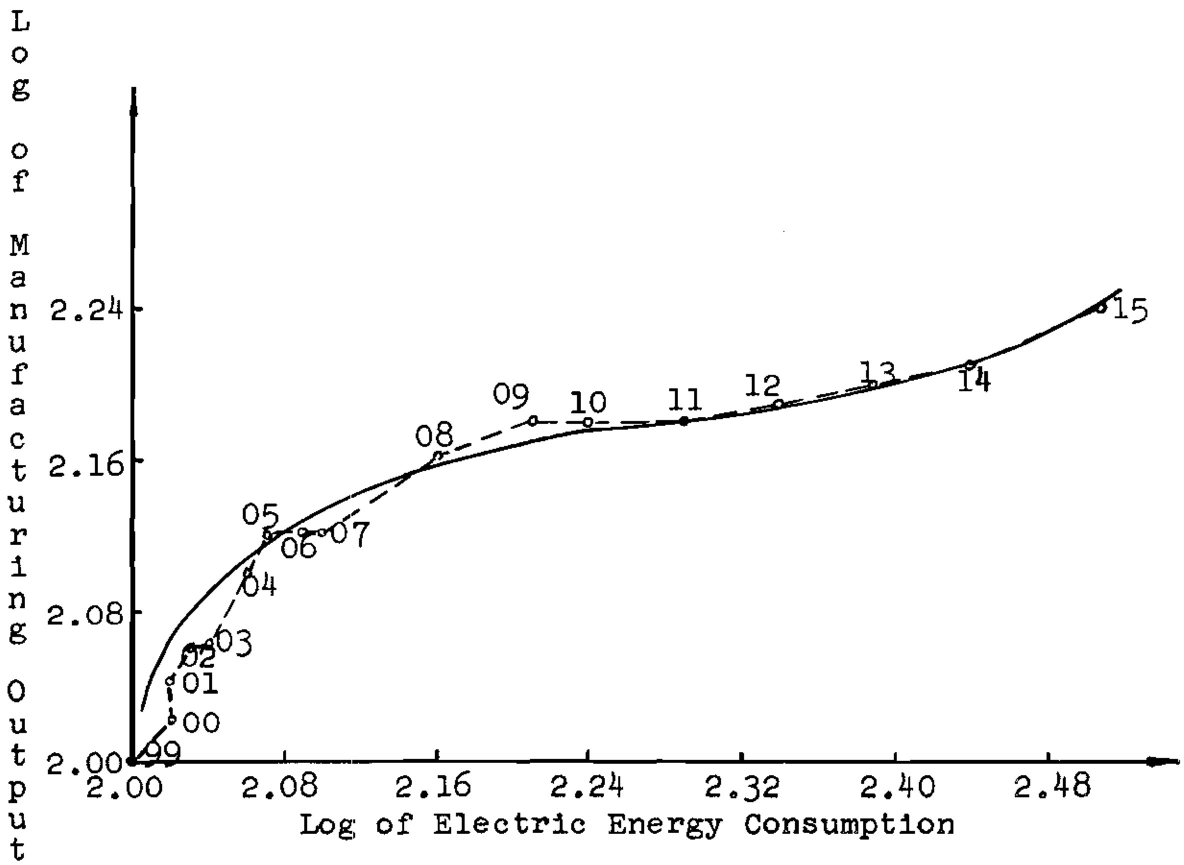


Figure 19. Relation of Manufacturing Output and Electric Energy Consumption: U.S.A.

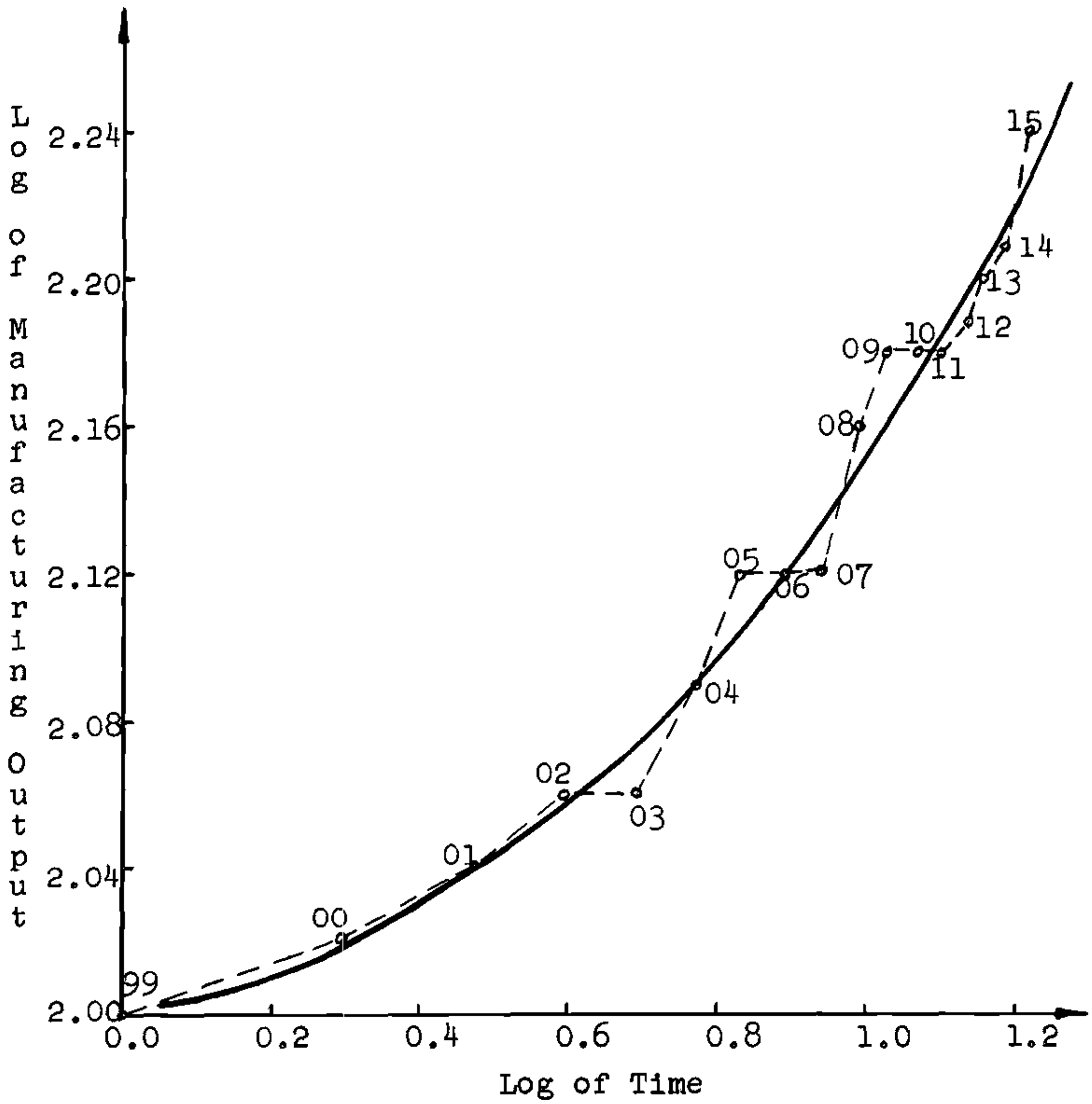


Figure 20. Relation of Manufacturing Output and Time: U.S.A.

Let

P = value of manufacturing output

C = value of capital investment

W = value of wages and salaries

R = value of raw materials

E = value of electrical energy or power

t = time (with 1939 for Mexico and 1899 for the U.S.A. as the origin).

We have seen that our input-output system may be defined as:

$$\text{Manufacturing Output} = f\,(X_1, X_2, \cdots X_n) \tag{21}$$

then

$$P = f\,(C, W, R, E, t) \tag{22}$$

for Models I (Mexico) and II (U.S.A.).

Our dynamic production function takes the hypothetical form:

$$P = aC^{\alpha} W^{\beta} R^{\lambda} E^{\delta} t^{\epsilon} \tag{23}$$

This function is basically of the Cobb-Douglas type. Some further reasons for our choice of such a function are:

(1) It yields production elasticities readily.

(2) The phenomenon of decreasing, increasing, or constant returns is exhibited with the use of the least complicated function.

(3) It is applicable to our index data, our multiple

inputs, and readily permits the study of changes in factors.

(4) Previous research found it to be adaptable to multiple regression techniques in the logarithms.

The two models specifically synthesized for each country with their foundation on the analysis of the factors as described in this section are:

(1) Model I - Mexico (1939 - 1955)

Let	$X_1 = \log P$	$X_5 = \log E$
	$X_2 = \log C$	$X_6 = Mt^*$
	$X_3 = \log W$	$X_7 = Mt^{2*}$
	$X_4 = \log R$	$k_o = \log a$
		$M = \log_{10} e^*$

We have then

$$\text{Log } P = \text{Log } a + \alpha \text{ Log } C + \beta \text{ Log } W + \lambda \text{ Log } R + \delta \text{ Log } E + \epsilon Mt + \phi Mt^2 \tag{24}$$

or

$$X_1 = k_o + \alpha X_2 + \beta X_3 + \lambda X_4 + \delta X_5 + \epsilon X_6 + \phi X_7 \tag{25}$$

and in the multiplicative form:

$$P = aC^{\alpha} W^{\beta} R^{\lambda} E^{\delta} e^{\epsilon t + \phi t^2} \tag{26}$$

*The methodology of Graphic Scaffolding as described by Schultz prescribes the use of these elements.

(2) Model II - U.S.A. (1899 - 1915)

Let $X_1 = \log P$ $\qquad X_6 = (\log E)^2$

$X_2 = \log C$ $\qquad X_7 = (\log E)^3$

$X_3 = \log W$ $\qquad X_8 = \log t$

$X_4 = \log R$ $\qquad X_9 = (\log t)^2$

$X_5 = \log E$ $\qquad k_o = \log a$

and if we recollect that the two modifications in this model as compared to Model I - Mexico are:

(a) $$\text{Log } P = f\ (\text{Log } E) = a + \text{Log } E + (\text{Log } E)^2 + (\text{Log } E)^3 \qquad (27)$$

(b) $$\text{Log } P = f\ (\text{Log } t) = a + \text{Log } t + (\text{Log } t)^2 \qquad (28)$$

we have

$$\text{Log } P = \text{Log } a + \propto \text{Log } C + \beta \text{ Log } W + \lambda \text{ Log } R + \delta \text{ Log } E + \epsilon\ (\text{Log } E)^2 + \phi(\text{Log } E)^3 + I \text{ Log } t + K(\text{Log } t)^2 \qquad (29)$$

or

$$X_1 = k_o + \propto X_2 + \beta X_3 + \lambda X_4 + \delta X_5 + \epsilon X_6 + \phi X_7 + IX_8 + KX_9 \qquad (30)$$

and in the multiplicative form:

$$P = aC^{\alpha} W^{\beta} R^{\lambda} E^{\delta} 10^{\epsilon(\text{Log } E)^2} 10^{\phi(\text{Log } E)^3} t^{I} 10^{K(\text{Log } t)^2} \tag{31}$$

We are now in a position to predict the total output curve or surface as a regression equation. The regression coefficients or elasticities are to be solved by means of multiple regresstion methods. The IBM 650 electronic computer was used for this purpose with the assistance of the Multiple Regression Sub-Routine ST-01. This sub-routine supplies the following information: (1) mean of each factor time series, (2) standard error of each factor, (3) partial correlation coefficients, (4) regression coefficients, (5) standard error of regression coefficients, (6) zero-order correlation matrix, and (7) predicted values for the endogenous variable in our regression model.

The models developed appear below:

<u>Model I - Mexico (1939 - 1955)</u>

$$X_1 = 0.43040 + 0.51456X_2 - 0.41493X_3 + 0.33686X_4 + 0.31929X_5 + 0.10140X_6 - 0.00372X_7 \tag{32}$$

or

$$P = 2.6940C^{0.51456} W^{-0.41493} R^{0.33686} E^{0.31929} e^{0.10140t-0.00372t^2} \tag{33}$$

Model II - U.S.A. (1899 - 1915)

$$X_1 = \bar{2}.86765 + 0.16195X_2 + 0.39537X_3 + 0.37697X_4 + 3.81074X_5 - 1.61273X_6 + 0.22857X_7 - 0.00718X_8 - 0.02027X_9 \qquad (34)$$

or

$$P = 0.00737C^{0.16195}\,W^{0.39537}\,R^{0.37697}\,E^{3.81074}\,10^{-1.61273(\text{Log }E)^2}\,10^{0.22857(\text{Log }E)^3}\,t^{-0.00718}\,10^{-0.02027(\text{Log }t)^2} \qquad (35)$$

The analysis of the models is considered in the next chapter.

CHAPTER V

ANALYSIS OF THE MODELS

The analysis of our models should be oriented so as to permit sound conclusions. Furthermore, we must keep in mind that our objective is to find which relevant factors came into play in the development and growth of manufacturing output in each country during the periods studied. This investigation is comparative insofar as we can isolate the basic differences as brought out by the model structures.

1. The goodness-of-fit of our output surface is estimated from the unbiased estimate of the value of the standard error.

$$\overline{S}^2_{1.2\cdots n} = \frac{N\,\sigma_z^2}{N - n} \tag{36}$$

where

$\overline{S}$ = unbiased standard error of the estimate

N = number of observations

n = total number of variables

σ_z^2 = variance of residuals

The closeness with which the estimated values agree with the original values is shown in Table 4.

2. To obtain the unbiased estimate of the multiple correlation coefficients we apply:

$$\bar{R}^2_{1.2\cdots n} = 1 - \left[\left(\frac{\bar{S}^2_{1.2\cdots n}}{\sigma_1^2} \right) \left(\frac{N - 1}{N} \right) \right] \tag{37}$$

where

$\bar{R}_{1.2\cdots n}$ = adjusted coefficient of multiple regression

σ_1^2 = standard error of the dependent variable

$$\bar{R}_{1.2\cdots n} = \sqrt{\bar{R}^2_{1.2\cdots n}} \tag{38}$$

This will give us the proportion of the variation in the dependent factor which can be explained by, or is associated with, variation in the independent factors. We may then test for the significance of this coefficient ($\bar{R}_{1.2\cdots n}$) and therefore that of the linear relationship existing in our regression equation. The ratio

$$F = \frac{\bar{R}^2_{1.2\cdots n}\ (N - n)}{(1 - \bar{R}^2_{1.2\cdots n})\ (n - 1)} \tag{39}$$

is Snedecor's "F" with (n - 1) and (N - n) degrees of freedom.

We test the null hypothesis

$$H_0: \bar{R}_{1.2\cdots n} = 0$$

with the alternate

$$H_1: \bar{R}_{1.2\cdots n} \neq 0$$

If we reject H_0, our multiple regression coefficient is significantly different from zero and there exists a linear relationship in our function, i.e., the goodness-of-fit obtained is adequate.

3. The square of the coefficient of multiple correlation, $\overline{R}^2$, is the coefficient of multiple determination which indicates that proportion of the variance in the dependent variable which has been mathematically accounted for. The calculated values of $\overline{R}$ and $\overline{R}^2$ are shown in Table 4.

4. The partial correlation coefficients (r_{1j}) are measures of the importance of each of the individual variables taken separately, while simultaneously allowing for the variation associated with the remaining independent variables. To put it in another manner, these coefficients are an indication of the extent to which that part of the variation in the dependent variable which was not explained by the other independent factors <u>can be explained</u> by the addition of the new factor. We thus obtain the relative imputed importance of each factor as related to value of manufacturing output. The calculated values of r_{1j} and r_{1j}^2 are shown in Table 5.

5. The regression coefficients (b_{1j}) are to be tested for significance to justify their existence, so-to-speak, within the hypothesized function. The test will be applied with a null hypothesis that in the population the regression coefficient is zero as follows:

Table 4. Multiple Coefficients for Regression Models

MODEL	DEP. VAR.	IND. VARIABLES	STD.ERROR $\overline{S}_{1.2\cdots n}$	COEFF.OF MULT. CORRELATION $\overline{R}$	COEFF. OF MULT. DETERM. $\overline{R}^2$
I MEX.	P	C,W,R,E,Mt,Mt^2	0.00998	0.99301	0.98604[a]
II U.S.A.	P	C,W,R,E,E^2,E^3,t,t^2	0.00062	0.99996	0.99992[b]

a Significantly different from zero at 1 per cent level

b Significantly different from zero at .1 per cent level

Table 5. Relative Importance of Individual Factors Affecting Value of Manufacturing Output as Indicated by Coefficient of Partial Correlation

MODEL I - MEX.			MODEL II - U.S.A.		
FACTOR[a] ADDED	COEFF.OF PART. CORRELA. (r_{1j})	REDUC.IN UNEXP. VARIANCE (r^2_{1j})	FACTOR[a] ADDED	COEFF.OF PART. CORRELA. (r_{1j})	REDUC.IN UNEXP. VARIANCE (r^2_{1j})
C	0.33	0.11	C	0.14	0.02
W	-0.20	0.04	W	0.28	0.08
R	0.16	0.03	R	0.50	0.25
E	0.36	0.13	E	0.48	0.66
Mt	0.53	0.73	E^2	-0.46	
Mt^2	-0.67		E^3	0.46	
			t	-0.21	0.35
			t^2	-0.56	

a All other factors are considered, but this is the factor added which directly influences the reduction in the unexplained variance of the dependent variable P by an amount equal to r^2_{1j}.

$$H_0: \quad \alpha = 0; \quad \beta = 0; \quad \lambda = 0, \text{ etc.}$$

where α, β, λ, etc., are the regression coefficients of the population.

The alternate hypothesis is:

$$H_1: \quad \alpha \neq 0; \quad \beta \neq 0; \quad \lambda \neq 0, \text{ etc.}$$

Formula

$$t_{1j} = \frac{b_{1j} - \beta_{1j}}{\Delta b_{1j}} \tag{40}$$

is used for the significance tests, where

Δb_{1j} = standard error of the regression coefficients.

The t_{1j} are distributed as the Student's "t" with (N - n) degrees of freedom.

If we should reject H_0, we will consider the regression coefficients as significant and can conclude that there probably is a linear relationship between X_1, the dependent variable, and X_j (where $j = 2, 3 \cdots n$), the independent variables in question. Our objective will be to find at what level they do become significant. Please refer to Tables 6 and 7.

From the standpoint of manufacturing policy, tests of significance indicate when a given factor of production is likely to influence the value or amount of the output.

Table 6. Model I - Mexico: Regression Coefficients (b_{1j}) and Their Standard Errors (Δb_{1j})

	CAPITAL	WAGES-SAL.	RAW MAT.	POWER	TIME	
	C X_2	W X_3	R X_4	E X_5	Mt X_6	Mt^2 X_7
REGRESSION COEFFICIENT	0.51456	-0.41493	0.33686	0.31929	0.10140	-0.00372
STANDARD ERROR	0.46044	0.62899	0.65013	0.25986	0.05074	0.00130
t-VALUES	1.11753 a	-0.65967 b	0.51814 c	1.22870 d	2.00000 e	-2.86153 f

a Significant at the 30 per cent level

b Significant at the 55 per cent level

c Significant at the 65 per cent level

d Significant at the 25 per cent level

e Significant at the 10 per cent level

f Significant at the 2 per cent level

Table 7. Model II - U.S.A.: Regression Coefficients (b_{1j}) and Their Standard Errors (Δb_{1j})

	CAPITAL	WAGES-SAL.	RAW MAT.	POWER			TIME	
	C	W	R	E	E	E	t	t^2
	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
REG.COEFF.	0.16195	0.39537	0.37697	3.81074	-1.61273	0.22857	-0.00718	-0.02027
STD.ERROR	0.38979	0.48638	0.23357	2.49705	1.08787	1.57985	0.01172	0.01065
t-VALUES	0.41548	0.81288	1.61394	1.52609	-1.48246	0.14467	-0.61262	-1.90328
	a	b	c	d	e	f	g	h

a Significant at the 70 per cent level

b Significant at the 45 per cent level

c Significant at the 15 per cent level

d Significant at the 20 per cent level

e Significant at the 20 per cent level

f Significant at the 90 per cent level

g Significant at the 55 per cent level

h Significant at the 10 per cent level

It is pertinent to point out that some of the statistical analyses used have a dubious validity when we deal with small samples in time series.

6. The elasticities of production with regard to each production factor or independent variable are investigated to determine the nature of returns accruing to each factor as the rest are held constant.

The derivation of the elasticities follows:

given

$$X_1 = k_o + \alpha X_2 + \beta X_3 \cdots\cdots + \epsilon X_n \qquad (41)$$

then

$$\frac{\partial X_1}{\partial X_j} = \frac{\partial \text{(Log of Output)}}{\partial \text{(Log of Input)}} = b_{1j} \qquad (42)$$

where

b_{1j} = regression coefficient or elasticity.

In investigating the nature of returns to each factor, we may inquire if we have constant, diminishing, or increasing returns to scale. The following conditions will hold for each case respectively:

if $b_{1j} = 1$

we have constant returns for factor X_j

$$\text{if} \quad b_{1j} < 1$$

we have diminishing returns for factor X_j

$$\text{if} \quad b_{1j} > 1$$

we have increasing returns for factor X_j

Along similar lines, the entire regression function is investigated for the characteristics of returns but taking all the primary factors in combination. The whole of our equation is analyzed producing an overall picture of manufacturing industry. The following conditions would be applicable:

$$\text{if} \sum_{j=2}^{n} b_{1j} = 1$$

we have total constant returns

$$\text{if} \sum_{j=2}^{n} b_{1j} < 1$$

total returns increase at a decreasing rate

$$\text{if} \sum_{j=2}^{n} b_{1j} > 1$$

total returns increase at an increasing rate

where $j = 2, \cdots n$.

The above analysis points to such aspects as:

(a) Emphasis on stressing low-return factors within the manufacturing sector.

(b) Creating an awareness for problem areas as shown by unexpected or malignant nature of returns.

(c) Indication for optimum use of resources.

(d) Overall view of the efficiency of our "plant"-- in this case manufacturing industry.

Before such analysis can take place, the model structures must be put in comparative form. The two models in the original form are as follows:

<u>Model I - Mexico</u>:

$$P = 2.6940C^{0.51456}\, W^{-0.41493}\, R^{0.33686}\, E^{0.31929}\, e^{0.10140t-0.00372t^2} \tag{43}$$

<u>Model II - U.S.A.</u>:

$$P = 0.00737C^{0.16195}\, W^{0.39537}\, R^{0.37697}\, E^{3.81074}\, 10^{-1.61273(\text{Log } E)^2+0.22857(\text{Log } E)^3}\, t^{-0.00718}\, 10^{-0.02027(\text{Log } t)^2} \tag{44}$$

The power factor (E) in Model II - U.S.A. should be modified to obtain an equation which will be comparable in relation to elasticities. Reference to Figure 19 (page 38) will show the product-factor relationship to be:

$$\text{Log } P = a + \text{Log } E + (\text{Log } E)^2 + (\text{Log } E)^3 \tag{45}$$

If we define the elasticity of production to be:

$$e = \frac{\text{percentage increase in output}}{\text{percentage increase in input}}$$

we can estimate the elasticity for the output - power input relationship. This may be exhibited by a series of "elasticity rays" as shown below in Figure 21.

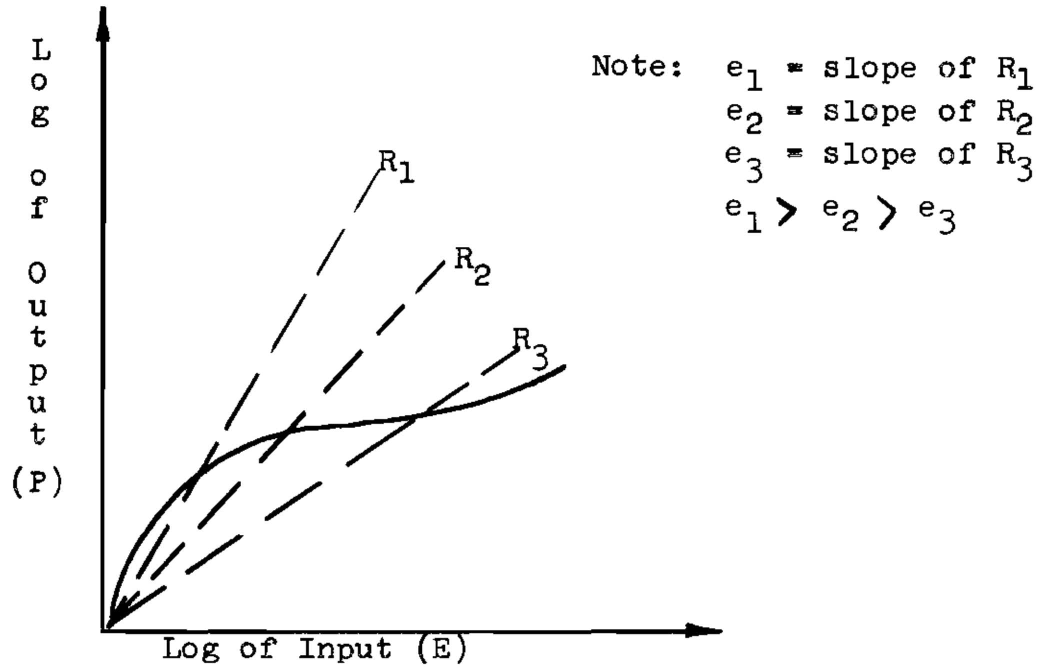


Figure 21. "Elasticity Rays" for Power Input

The minimum elasticity, i.e., the worst possible condition, is the ray passing through the observations for the year 1915 in our data for power inputs. The elasticity for power input under these conditions is:

$$e = \frac{2.23553}{2.51188} = 0.88996$$

In comparative form, the modified U.S.A. model can be stated as follows:

<u>Model II - U.S.A. (modified)</u>:

$$P = 0.00737C^{0.16195}\,W^{0.39537}\,R^{0.37697}\,E^{0.88996}\,t^{-0.00718}\,10^{-0.02027(\text{Log } t)^2} \qquad (46)$$

Tables 8 and 9 show the nature of returns for individual and combined use of productive factors.

Table 8. Nature of Returns for Individual Production Factors

MODEL I - MEX.				MODEL II - U.S.A. (Mod.)			
$b_{1j} < 1$	$b_{1j} = 1$	$b_{1j} > 1$	Other	$b_{1j} < 1$	$b_{1j} = 1$	$b_{1j} > 1$	Other
C				C			
			W	W			
R				R			
E				E			

Table 9. Nature of Returns for Combined Use of Productive Factors [a]

MODEL I - MEX.	MODEL II - U.S.A. (Mod.)
$\sum_{j=2}^{n} b_{1j} = 0.75578$	$\sum_{j=2}^{n} b_{1j} = 1.80425$

a This implies the characteristic of returns for the entire function, i.e., the whole of manufacturing industry.

7. We can investigate the marginal characteristics of our power function for Mexico (Model I - Mexico) in the following manner:

given the function

$$P = aC^{\alpha} W^{\beta} R^{\lambda} E^{\delta} e^{\epsilon t + \phi t^2}$$

(a) The marginal productivity (MP) of C is given by

$$\frac{\partial P}{\partial C} = c \alpha C^{\alpha - 1} \tag{47}$$

where

$$aW^{\beta} R^{\lambda} E^{\delta} e^{\epsilon t + \phi t^2} = \text{constant} = c$$

(b) The (MP) of W is

$$\frac{\partial P}{\partial W} = d \beta W^{\beta - 1} \tag{48}$$

where

$$aC^{\alpha} R^{\lambda} E^{\delta} e^{\epsilon t + \phi t^2} = \text{constant} = d$$

(c) The (MP) of R is

$$\frac{\partial P}{\partial R} = h \lambda R^{\lambda - 1} \tag{49}$$

where

$$aC^{\alpha} W^{\beta} E^{\delta} e^{\epsilon t + \phi t^2} = \text{constant} = h$$

(d) The (MP) of E is

$$\frac{\partial P}{\partial E} = k\,\delta\,E^{\delta-1} \tag{50}$$

where

$$aC^{\alpha}\,W^{\beta}\,R^{\lambda}\,e^{\epsilon t+\phi t^2} = \text{constant} = k$$

The constants c, d, h, and k are evaluated at the geometrical means of the inputs for a specific year t. We are then able to estimate whether we have been using too much or not enough of a specific resource if we subject our (MP) values to the following conditions:

Let

$(MP)_j$ = marginal productivity of factor "j",

p_j = market price of factor "j",

then if

$$(MP)_j > p_j \tag{51}$$

more of this resource should be used.

If

$$(MP)_j < p_j \tag{52}$$

smaller quantities of this resource should be employed.

The p_j values are difficult to obtain for aggregate investigations, but with basic data, weighting for amount of material or wage earners in specific sectors, price levels approximations to actual p_j's may be obtained.

8. As a last point of analysis, the simple correlation coefficients r_{ij} (where i, j = 1, 2 ··· n) are tabulated in Tables 10 and 11 to demonstrate the degree of correlation between pairs of variables or factors.

Discussion of the models, based on the analytical methods outlined in this chapter, is considered in the next chapter.

Table 10. Zero-Order Correlation Matrix for Model I - Mexico, 1939 - 1955

	Value of Mfg. Output X_1	Total Cap. Invest. X_2	Wages & Salaries X_3	Raw Mat. Consump. X_4	Power Consump. X_5	Time X_6	Time X_7
X_1	1.00	-0.67	0.39	0.88	0.81	0.81	0.68
X_2		1.00	0.00	-0.58	-0.77	-0.95	-0.92
X_3			1.00	0.71	0.56	0.28	0.32
X_4				1.00	0.86	0.78	0.72
X_5					1.00	0.92	0.92
X_6						1.00	0.97
X_7							1.00

Table 11. Zero-Order Correlation Matrix
for Model II - U.S.A., 1899 - 1915

	Value of Mfg.Output	Total Cap. Invest.	Wages & Salaries	Raw Mat. Consump.	Power Consumption			Time	
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
X_1	1.00	0.99	0.99	0.99	0.90	0.89	0.88	0.96	0.99
X_2		1.00	0.99	0.99	0.89	0.88	0.87	0.97	0.99
X_3			1.00	0.99	0.89	0.88	0.87	0.96	0.99
X_4				1.00	0.90	0.89	0.88	0.96	0.99
X_5					1.00	0.99	0.99	0.80	0.91
X_6						1.00	0.99	0.79	0.90
X_7							1.00	0.78	0.89
X_8								1.00	0.96
X_9									1.00

CHAPTER VI

DISCUSSION OF THE MODELS

The pertinent observations on analysis of the results are enumerated below.

1. The goodness-of-fit of our output surface is, in our estimation, fair for Model I - Mexico, and very good for Model II - U.S.A., as indicated by the standard error of the estimate $\overline{S}_{1.2..n}$. We can observe from Table 12 the actual and predicted values for the value of manufacturing output and further judge our accuracy. The standard error of our estimate $\hat{P}$ is approximately five points for Model I - Mexico; whereas, on the other hand, the error is negligible (approximately 0.1 points) in Model II - U.S.A.

2. We have been able to explain practically all the variation in the dependent factor associated with the variations in the independent variables. As indicated by the coefficient of determination, we have accounted mathematically for 99 per cent of the variance in our output factor (refer to Table 4) in our models.

3. The significance test for the linear relationship in our regression equation is highly significant and such a relationship clearly exists for both models.

4. As each specific factor is added to our function, we can reduce the unexplained variance in our output and

Table 12. Comparison Between The Actual Index of Production (P) and the Estimated Index ($\hat{P}$) for Both Models

Model I - Mexico (1939 - 1955)					Model II - U.S.A. (1899 - 1915)				
Year	Actual Output Index P	Est. Index $\hat{P}$	Dev. $(\hat{P} - P)$	Per Cent Dev. $\frac{\hat{P} - P}{P}$	Year	Actual Output Index P	Est. Index $\hat{P}$	Dev. $(\hat{P} - P)$	Per Cent Dev. $\frac{\hat{P} - P}{P}$
1939	100	97	-3.0	-3.0	1899	100	100	--	--
1940	111	115	4.0	3.6	1900	104	104	--	--
1941	134	139	5.0	3.7	1901	109	109	--	--
1942	152	152	--	--	1902	114	114	--	--
1943	158	149	-7.0	-4.4	1903	116	116	--	--
1944	151	151	--	--	1904	124	124	--	--
1945	164	167	3.0	1.8	1905	131	131	--	--
1946	168	159	-9.0	-5.4	1906	133	133	--	--
1947	162	170	8.0	4.9	1907	132	132	--	--
1948	174	179	5.0	2.9	1908	145	145	--	--
1949	178	185	7.0	3.9	1909	152	152	--	--

Table 12. (Continued)

Model I - Mexico (1939 - 1955)					Model II - U.S.A. (1899 - 1915)				
Year	Actual Output Index P	Est. Index P	Dev. (P - P)	Per Cent Dev. $\frac{P - P}{P}$	Year	Actual Output Index P	Est. Index P	Dev. (P - P)	Per Cent Dev. $\frac{P - P}{P}$
1950	206	200	-6.0	-2.9	1910	150	150	--	--
1951	179	179	--	--	1911	152	152	--	--
1952	178	175	-3.0	-1.7	1912	156	156	--	--
1953	204	195	-9.0	-4.4	1913	158	158	--	--
1954	199	196	-3.0	-1.5	1914	161	162	+1.0	+0.6
1955	159	166	7.0	4.4	1915	172	172	--	--

gauge the importance of the "added" input (refer to Table 5). The reduction in unexplained variance due to each factor, in order of decreasing importance, is as follows:

Table 13. Importance of Each Factor in Reducing The Unexplained Variance in The Output

(a) Model I - MEX.	(b) Model II - U.S.A.
Time	Power Consumption
Power Consumption	Time
Capital	Raw Materials
Wages and Salaries	Wages and Salaries
Raw Materials	Capital

For these models it seems that time and power consumption inputs explain a larger proportion of the variation associated with the dependent variable or output. The practical implications are deferred until discussion of the regression coefficients (elasticities).

5. The regression coefficients or production elasticities as they may be called (b_{ij}) are presented in Tables 6 and 7 for Models I - Mexico and II - U.S.A., respectively. From the standpoint of economic policy, we are interested in obtaining an indication of how much a given factor of production is likely to influence the amount of the product.

The elasticities give us such an indication. The standard errors of each are included to clarify their accuracy within a probability level of 68 per cent.

We are told that negative elasticities may occur but are not statistically significant.[9] There is an interesting result as exemplified by the significance test performed on each elasticity. Second-degree power consumption, for example, has a negative exponent yet it becomes significant at the lower levels. Such tests, of course, preclude our assumption of normality and independence in the errors or deviations of our data. This may be a dubious assumption.

Table 14. Significance Levels of Regression Coefficient for Both Models

Model	Level at Which Regression Coeff. Becomes Significant								
	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90
I MEX.	X_7	X_6	X_2, X_5	-	-	X_3	X_4	-	-
II U.S.A.	X_9	X_4, X_5, X_6	-	-	X_3	X_8	X_2	-	X_7

For Model I-MEX.

X_2=Log C
X_3=Log W
X_4=Log R
X_5=Log E
X_6=Mt
X_7=Mt^2

For Model II-U.S.A.

X_2=Log C
X_3=Log W
X_4=Log R
X_5=Log E
X_6=(Log E)^2
X_7=(Log E)^3
X_8=Log t
X_9=(Log t)^2

From Table 14, indications are that we could drop wages and salaries, and raw materials from Model I - Mexico, and do the same with wages and salaries, capital, and time from Model II - U.S.A. This does not seem to be logical and we must temper statistical analysis of our results with practical economic reasoning.

The negative elasticity for wages and salaries in Model I - Mexico is not to be interpreted literally. It should be observed as a result of the interrelationships existing and changes occurring in all the variables within our regression problem.

With regard to combined input, we found when testing the significance of the overall goodness-of-fit ($\bar{R}_{1.2\ldots n}$) that our multiple regression coefficient was highly significant. However, when each independent factor is taken separately, the linear relationship between each and the output is greatly to be doubted as is indicated by our factor significant tests.

6. An analysis of the elasticities places the basic factors in the following order of importance for each model:

Table 15. Importance of Factor Elasticities for Both Models

(a) Model I - Mexico	(b) Model II - U.S.A.
C (Capital)	E (Power)
R (Raw Materials)	W (Wages and Salaries)
E (Power)	R (Raw Material)
W (Wages and Salaries)	C (Capital)

We can expect greater percentage increases in the value of output from capital, raw materials, and wages and salaries inputs (in that order) for Model I - Mexico, *ceteris paribus*. This seems a logical result since industrial product prices reflect an interest on the part of industry to cover input values or costs in order of magnitude of factor prices. In Mexican manufacturing industry a factor-price classification of the variables involved would closely follow the sequence in Table 15 (a).

Venture capital for industrial investment is a difficult (1939 - 1955) resource to acquire and its consequent cost or factor price is high. This is aggravated by the costly import of capital goods, equipment and accessories. Furthermore, except for the years 1943-1946 approximately, there is much evidence to indicate excess capacity conditions in many branches of manufacturing. Import prices and quantities of required raw materials (as well as high prices of domestically available materials) contribute to placing this factor in a prominent position on the scale. Power rates are not known to be low in Mexico; the corresponding input value seems to be higher than wages and salaries (our "insignificant" factor). Profit levels are concealed in this study by value of output data. Levels are high and create disturbances affecting participation of all factors, primarily wages as has been mentioned previously.

In the U.S.A., value of power rates added to the intensive use of capacity make this factor highly important in its effects on value of output. The value of labor plays a more logical role in the setting of manufacturing industry. The great upswing of labor costs is reflected by the value of our wages and salaries factor and its effect on the value of output. Raw materials follow closely and is to be expected from the period studied as manufacturing methods and processes added to costs in the extractive industries determine this factor's importance. Domestic capital goods industries were already active in the U.S.A. in the first twenty years of the century, and there was great fluidity in capital for investment purposes. These conditions reduced the relative value position of capital in relation to other input factors and therefore its influence on the output value increment.

An analysis of the "physical" production aspects of our functions show the following observations for Model I - Mexico:

(a) Decreasing returns accrue to each of the factors involved.

(b) Increasing costs per unit output for each factor considered.

(c) The overall model describes conditions of returns increasing at a decreasing rate.

Some implications are that the capital-intensive nature of manufacturing development in Mexico for 1939 - 1955 has sped up growth but has lagged behind in the efficient use of existing and new equipment installations. Quality and processing of raw materials and manufactured product with the purpose of reducing costs of materials and waste have not been sufficiently stressed. Manufacturing processes must be used optimally for minimizing the costs of production.

In Model II - U.S.A., conditions are similar except for the power consumption factor.

7. As a final point, we may investigate the characteristics of the dynamic factors in our models, namely:

For Model I - Mexico:

$$f(t) = e^{0.10140t-0.00372t^2} \tag{53}$$

For Model II - U.S.A.:

$$f(t) = t^{-0.00718}\ 10^{-0.02027(\mathrm{Log}\ t)^2} \tag{54}$$

The dynamics of growth and the effect of some of the other factors not included in our models may then be observed from Figure 22. It is interesting to note that during the same interval of seventeen years, the value of f(t) for the Mexican model increased from 1.10 to 1.99, while for the U.S.A. model, the value of f(t) remained relatively constant at the level of approximately 0.95. These results are not surprising because we have already seen that the Model I - Mexico is not as exact as the Model II - U.S.A. We can observe that some causal factors of importance have not been included in the former model.

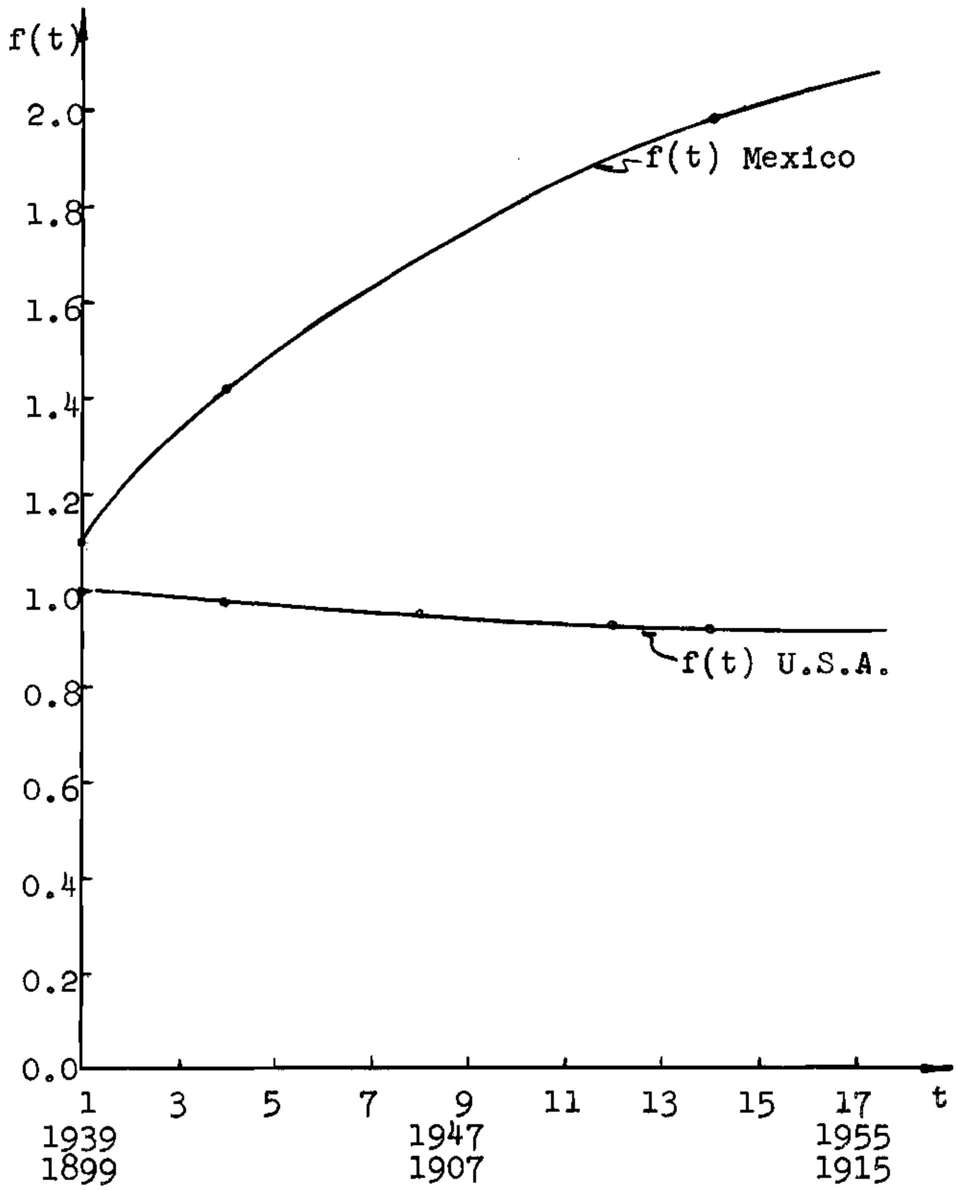


Figure 22. Dynamic Factors for
Models I - Mexico and II - U.S.A.

CHAPTER VII

CONCLUSIONS

It has been our purpose to quantify a portion of the economic phenomena underlying the growth of manufacturing industry in Mexico and the U.S.A. during 1939 - 1955 and 1899 - 1915 respectively. Throughout the exposition attention was drawn to the possibilities open to the investigator for the formulation of planning criteria that evolve from such analysis. As is to be expected, estimates and projections can only be as accurate as the data available and the degree of sophistication in the analytical tools employed.

Our regression functions have approximated some possible "laws" governing manufacturing output for the periods studied. They provide a basis for measuring past effort, and to a limited extent enable us to plan for development of manufacturing industry in Mexico.

Some of the main points brought out by this study are summarized below:

1. Two models, each based on four primary factors (C = capital, W = wages and salaries, R = raw materials, E = power consumption) and a time factor t, were constructed for the manufacturing industries of Mexico and the U.S.A.

respectively. In a comparative form, the two models are as follows:

Model I - Mexico:

$$P = 2.6940C^{0.51456} W^{-0.41493} R^{0.33686} E^{0.31929} e^{0.10140t-0.00372t^2} \tag{55}$$

for the period 1939 - 1955.

Model II - U.S.A.:

$$P = 0.0074C^{0.16195} W^{0.39537} R^{0.37697} E^{0.88996} t^{-0.00718} 10^{-0.02027(\text{Log } t)^2} \tag{56}$$

for the period 1899 - 1915.

2. The returns of manufacturing industry in Mexico were increasing at a decreasing rate for the period 1939 - 1955. This observation is based on the fact that the sum of the elasticities for the primary inputs is 0.756.

3. The returns of manufacturing industry in the U.S.A. were increasing at an increasing rate for the period 1899 - 1915. In this case, the sum of the elasticities of primary inputs is 1.804.

4. Comparison of the two models shows that the rate of capital investment in Mexico is about three times higher in relation to the same factor in the U.S.A. Also, the effect of the raw materials input is about the same in both models.

5. Some weaknesses of the structure of the Mexican manufacturing industry are clearly illustrated by the nature of the following two factors:

(a) The most critical area is the one of labor input, W, which has a negative coefficient of elasticity -0.415.

(b) The power consumption input is almost one third of the corresponding factor in the U.S.A. model.

6. A comparison of the factors C and W for the Mexican model shows the predominance of capital (0.515) over labor (-0.415). There has been an emphasis on capital investment with a serious lag in productivity. This may indicate that factors such as technical experience, management and administrative techniques have not been emphasized enough to cope with the advanced technology implied by such investment.

7. The sum of the elasticities for C and E in the Mexican model is 0.834, while for the U.S.A. model it is 1.052. The order of magnitude is similar, and it may be concluded that in Mexico the factor C has been stressed again at the expense of factor E.

8. The time factor has been very important in the Mexican model demonstrating the dynamic nature of development of manufacturing industry in that country. As time is

not a causal factor, it points to the necessity of expanding this model to include other significant variables affecting output in Mexican manufacturing industry.

APPENDIX

Table 16. Net Income Produced per Man-Hour in Manufacturing (in I. U.'s)[a]

YEAR	U.S.A.	YEAR	MEXICO
1889-1890	0.292	1948	0.486
1900-1904	0.220	1949	0.541
1913	0.356	1950	0.593

Source: C. Clark, The Conditions of Economic Progress, 3rd edition, London, Macmillan and Co., Ltd., 1957. Table VIII, p. 336.

[a]One I. U. of real income was taken as the quantity of goods exchangeable in the U.S.A. for $1 over the average of the decade 1925-1934.

Table 17. Income Produced Per Worker in Mexico for Manufacturing Industry

Year	Nat'l. Income[a] Prod. by Mfg. Ind. (millions of pesos)	Purchasing[b] Power Index 1939=100	Deflated National Income	Employed[c] in Mfg. Ind.	Mfg. Income Prod. Per Empl. Worker	Index of Income Produced Per Worker
1939	914.1	100.0	914.1	605,000	1510	100.0
1940	1027.3	97.2	998.5	639,607	1560	103.3
1941	1257.1	94.5	1188.0	673,000	1765	116.9
1942	1616.2	82.6	1335.0	704,000	1896	125.6
1943	2092.8	61.9	1295.4	735,000	1762	116.7
1944	2818.2	50.5	1423.2	768,000	1853	122.7
1945	3437.2	45.7	1570.8	801,000	1961	129.9
1946	4605.8	37.3	1718.0	832,000	2064	136.7
1947	4906.4	33.7	1653.4	864,000	1913	126.7
1948	5168.7	31.6	1633.3	895,000	1824	120.8
1949	5473.6	28.2	1543.6	929,000	1661	110.0

Table 17. (Continued)

Year	Nat'l. Income[a] Prod. by Mfg. Ind. (millions of pesos)	Purchasing[b] Power Index 1939=100	Deflated National Income	Employed[c] in Mfg. Ind.	Mfg. Income Prod. Per Empl. Worker	Index of Income Produced Per Worker
1950	6964.9	25.9	1803.9	972,545	1854	122.8

Sources: [a]*El Desarrollo Economico de Mexico*, Comision Mixta, Primera Edicion, Mexico D.F., Fondo de Cultura Economica, 1953, p. 36.

[b]*Memoria de la Secretaria de Economia*, Mexico D.F., 1958, p. 187.

[c]*Anuario Estadistico de los Estados Unidos Mexicanos*, 1940.
Anuario Estadistico de los Estados Unidos Mexicanos, 1945.
Anuario Estadistico de los Estados Unidos Mexicanos, 1950.

Table 18. Income Produced Per Worker in the U.S.A.
for Manufacturing Industry

Year	Nat'l. Income Prod. by Mfg. Ind. (millions of dls.)	Purchasing Power Index 1890-99=100	Deflated National Income	Employed in Mfg. Ind.	Mfg. Income Prod. Per Empl. Worker	Index of Income Produced Per Worker
1899	2.714	98	2.660	4,501,919	590	100.0
1900	2.941	94	2.764	4,630,000	596	101.0
1901	3.193	92	2.938	4,750,000	618	104.7
1902	3.605	90	3.244	4,835,000	670	113.6
1903	3.812	86	3.278	5,000,000	655	111.0
1904	3.519	87	3.062	5,181,660	590	100.0
1905	4.032	87	3.508	5,340,000	656	111.2
1906	4.377	84	3.677	5,552,000	662	112.2
1907	4.743	79	3.747	5,754,000	651	110.3
1908	4.046	83	3.358	6,025,000	557	94.4
1909	4.824	83	4.004	6,262,242	639	108.3
1910	5.447	78	4.249	6,315,000	672	113.9

Source: <u>Historical Statistics of the U.S.A.</u>, U. S. Department of Commerce, Series A 154-164, p. 14.

Table 19. Bank Loans and Discounts in Mexico

Year	Bank Loans and Discounts (millions of pesos)	Deflated to 1939 (pesos)	"Credit" Index (1939=100)
1939	237.0	237.0	100.0
1940	238.0	231.3	97.5
1941	312.0	294.8	124.5
1942	436.0	360.1	151.9
1943	606.0	375.1	158.2
1944	918.0	463.6	195.8
1945	1149.0	525.1	221.5
1946	1350.0	503.6	212.7
1947	1155.0	389.2	164.1
1948	1924.0	608.0	256.5
1949	1800.0	507.6	214.4
1950	1765.0	457.1	192.8

Source: La Economia Mexicana en 1953, Secretaria de Economia, Mexico D.F., 1954, p. 115.

Table 20. Bank Loans and Discounts in U.S.A.

Year	Bank Loans and Discounts (billions of dls.)	Deflated to 1890-1899 (dollars)	"Credit" Index 1899=100
1899	5.178	5.074	100.0
1900	5.658	5.318	104.8
1901	6.425	6.279	123.7
1902	7.189	6.470	127.5
1903	7.739	6.656	131.2
1904	7.932	6.944	136.9
1905	9.027	7.940	156.5
1906	9.894	8.983	177.0
1907	10.764	8.504	167.6
1908	10.438	8.664	170.8
1909	11.447	9.501	187.2
1910	12.522	9.767	192.5

Source: Historical Statistics of the United States, U. S. Department of Commerce, Series N 19-26, p. 262.

Table 21. Physical Volume of Production for Specific Manufacturing Industries in Mexico (1939=100)

Year	Textiles	Food and Beverages	Tobacco	Construction	Chemical Products	Iron and Steel	Leather Products
1939	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1944	129.5	149.6	113.9	206.0	148.4	117.1	180.8
1949	141.2	161.6	128.8	326.2	179.4	234.8	235.2
1954	167.1	234.2	146.3	474.1	278.2	339.2	n.d.

Source: Memoria de la Secretaria de Economia, Mexico D.F., 1955, p. 283-86.

Table 22. Physical Volume of Production for
Specific Manufacturing Industries in the U.S.A. (1899=100)

Year	Textiles	Food and Beverages	Tobacco	Construction	Chemical Products	Iron and Steel	Leather Products
1899	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1904	126.3	127.7	123.3	92.6	133.3	130.3	115.6
1909	157.8	150.0	143.3	101.2	188.8	225.1	129.6
1914	189.4	177.7	176.6	100.0	244.4	220.9	126.5

Source: Historical Statistics of the United States, U. S. Department of Commerce, Series J 15-29, p. 180.

Table 23. Percentage Distribution of Factors
in National Income: Mexico

Year	Agri-culture	Mining	Mfg.	Const.	Transp. & Utilities	Trade	Service	Gov't.	Finance & Misc.
1939-1944	19.1	5.1	16.9	1.8	6.1	26.4	8.3	6.0	9.1
1945-1950	17.6	3.9	18.3	1.9	4.8	31.9	7.8	4.4	8.3

Source: Comision Mixta, El Desarrollo Economico de Mexico, Primera Edicion, Mexico D.F., Fondo de Cultura Economica, 1953, p. 36.

Table 24. Percentage Distribution of Factors
in National Income: U.S.A.

Year	Agri-culture	Mining	Mfg.	Const.	Transp. & Utilities	Trade	Service	Gov't.	Finance & Misc.
1899-1908	16.7	3.1	18.4	4.5	10.7	15.3	9.6	5.6	16.0
1909-1918	17.7	3.3	20.8	3.2	10.7	14.5	8.2	6.3	15.4

Source: Historical Statistics of the United States, U. S. Department of Commerce, Series A 145-153, p. 13.

Table 25. Gross Value of Manufactured Product: Mexico

Year	Pesos[a] (millions)	Value in 1939 Pesos	Index 1939=100	Log of Index
1939	2113	2113	100	2.00000
1940	2421	2353	111	2.04532
1941	2999	2834	134	2.12710
1942	3890	3213	152	2.18184
1943	5392	3338	158	2.19866
1944	6314	3189	151	2.17898
1945	7583	3465	164	2.21484
1946	9514	3549	168	2.22531
1947	10161	3424	162	2.20952
1948	11642	3679	174	2.24055
1949	13328	3758	178	2.25042
1950	16794	4350	206	2.31387
1951	18000	3780	179	2.25285
1952	19900	3761	178	2.25042
1953	21730	4302	204	2.30963
1954	23580	4197	199	2.29885
1955	25430	3357	159	2.20140

Source: [a]*United Nations Yearbook*, 1948, Table 64, p. 169.
United Nations Yearbook, 1957, Table 68, p. 191.

Table 26. Capital Investment in Manufacturing Industry: Mexico

Year	Pesos[a] (millions)	Value in 1939 Pesos	Index 1939=100	Log of Index
1939	2375	2375	100	2.00000
1940	2650	2576	108	2.03342
1941	2950	2788	117	2.06819
1942	3280	2709	114	2.05690
1943	3640	2253	95	1.97772
1944	4000	2020	85	1.92942
1945	4365	1995	85	1.92942
1946	4750	1772	75	1.87506
1947	5150	1736	73	1.86332
1948	5550	1754	74	1.86923
1949	5965	1682	71	1.85126
1950	6395	1656	70	1.84510
1951	6800	1428	60	1.77815
1952	7200	1361	57	1.75587
1953	7675	1520	64	1.80618
1954	8160	1452	61	1.78533
1955*	8620	1138	48	1.68124

Source: [a]M. German Parra, La Industrializacion de Mexico, Mexico D.F., Imprenta Universitaria, 1954, p. 87.

* Estimated.

Table 27. Wages and Salaries
in Manufacturing Industry: Mexico

Year	Pesos[a] (millions)	Value in 1939 Pesos	Index 1939=100	Log of Index
1939	520	520	100	2.00000
1940	610	593	114	2.05690
1941	700	662	127	2.10380
1942	796	658	126	2.10037
1943	900	557	107	2.02938
1944	1050	530	102	2.00860
1945	1159	530	102	2.00860
1946	1350	504	97	1.98677
1947	1570	529	102	2.00860
1948	1850	585	112	2.04922
1949	2175	613	118	2.07188
1950	2490	645	124	2.09342
1951	2800	588	113	2.05308
1952	3135	592	114	2.05690
1953	3480	689	132	2.12057
1954	3790	675	130	2.11394
1955*	4120	544	105	2.02119

Source: [a]M. German Parra, La Industrializacion de Mexico, Mexico, D.F., Imprenta Universitaria, 1954, p. 87.

* Estimated.

Table 28. Raw Material Consumption in Manufacturing Industry: Mexico

Year	Pesos[a] (millions)	Value in 1939 Pesos	Index 1939=100	Log of Index
1939	1050	1050	100	2.00000
1940	1200	1166	111	2.04532
1941	1420	1342	128	2.10721
1942	1650	1363	130	2.11394
1943	1975	1222	116	2.06446
1944	2390	1207	115	2.06070
1945	2845	1300	124	2.09342
1946	3385	1263	120	2.07918
1947	4000	1348	128	2.10721
1948	4750	1501	143	2.15534
1949	5500	1551	148	2.17026
1950	6300	1632	155	2.19033
1951	7050	1480	141	2.14922
1952	7850	1484	141	2.14922
1953	8635	1710	163	2.21219
1954	9425	1678	160	2.20412
1955*	10200	1346	128	2.10721

Source: [a]M. German Parra, La Industrializacion de Mexico, Mexico, D.F., Imprenta Universitaria, 1954, p. 87.

*Estimated.

Table 29. Electrical Energy Consumption in Manufacturing Industry: Mexico

Year	Millions[a] of KWH	Price[b] cts/KWH	Cost of Energy in Pesos (thousands)	Deflated Cost of Energy 1939=100	Cost Index for Energy 1939=100	Log of Index
1939	415	3.8	15770	15770	100	2.00000
1940	475	4.2	19950	19390	123	2.08991
1941	524	4.7	24628	23280	148	2.17026
1942	580	5.2	30160	24910	158	2.19866
1943	639	5.9	37701	23340	148	2.17026
1944	695	6.5	45175	22820	145	2.16137
1945	760	7.0	53200	24310	154	2.18752
1946	836	6.9	57684	21510	136	2.13354
1947	913	7.9	72127	24310	154	2.18752
1948	941	8.6	80926	25570	162	2.20952
1949	1070	9.7	103790	29270	186	2.26951
1950	1197	11.9	142443	36890	234	2.36922
1951	1341	11.2	150192	31540	200	2.30103

Table 29. (Continued)

Year	Millions[a] of KWH	Price[b] cts/KWH	Cost of Energy in Pesos (thousands)	Deflated Cost of Energy 1939=100	Cost Index for Energy 1939=100	Log of Index
1952	1440	12.0	172800	32660	207	2.31597
1953	1555	13.0	202150	40030	254	2.40483
1954	1862	13.9	258818	46160	293	2.46687
1955	2099	14.9	312751	41280	262	2.41830

Sources: [a]El Caso de Mexico, Naciones Unidas, Consejo Economico Social, La Paz, Bolivia, Comision Economica Para America Latina, 1957, Vol. II, p. 289.

[b]Lara Beautell, C., La Industria de Energia Electrica, Primera Edicion, Mexico D.F., Fondo de Cultura Economica, 1953, p. 175.

Table 30. Time Series for
Model I - Mexico

Year	Time	M x Time*	$(\text{Time})^2$	M x $(\text{Time})^2$
1939	1	0.43430	1	0.43430
1940	2	0.86860	4	1.73720
1941	3	1.30290	9	3.90870
1942	4	1.73720	16	6.94880
1943	5	2.17150	25	10.85750
1944	6	2.60580	36	15.63480
1945	7	3.04010	49	21.28070
1946	8	3.47440	64	27.79520
1947	9	3.90870	81	35.17830
1948	10	4.34300	100	43.43000
1949	11	4.77730	121	52.55030
1950	12	5.21160	144	62.53920
1951	13	5.65090	169	73.39670
1952	14	6.08020	196	85.12280
1953	15	6.51450	225	97.71750
1954	16	6.94880	256	111.18080
1955	17	7.38310	289	125.51270

*$M = \text{Log}_{10}\ e = 0.43430$

Table 31. Gross Value of
Manufactured Product: U.S.A.

Year	Dollars[a] (billions)	Value in 1899 Dollars	Index 1899=100	Log of Index
1899	11.1	10.9	100	2.00000
1900	12.0	11.3	104	2.01703
1901	12.9	11.9	109	2.03743
1902	13.8	12.4	114	2.05690
1903	14.7	12.6	116	2.06446
1904	15.5	13.5	124	2.09342
1905	16.4	14.3	131	2.11727
1906	17.3	14.5	133	2.12385
1907	18.2	14.4	132	2.12057
1908	19.1	15.8	145	2.16137
1909	20.0	16.6	152	2.18184
1910	20.9	16.3	150	2.17609
1911	21.8	16.6	152	2.18184
1912	22.7	17.0	156	2.19312
1913	23.5	17.2	158	2.19866
1914*	24.4	17.6	161	2.20683
1915*	25.3	18.7	172	2.23553

Source: [a]Abstract of the Census of Manufactures, U. S. Department of Commerce, 1914, p. 16.

*Estimated.

Table 32. Capital Investment in Manufacturing Industry: U.S.A.

Year	Dollars[a] (billions)	Value in 1899 Dollars	Index 1899=100	Log of Index
1899	8.6	8.43	100	2.00000
1900	9.5	8.93	106	2.02531
1901	10.5	9.66	115	2.06070
1902	11.4	10.26	122	2.08636
1903	12.3	10.58	126	2.10037
1904	13.3	11.57	137	2.13672
1905	14.2	12.35	146	2.16435
1906	15.2	12.77	151	2.17898
1907	16.1	12.72	151	2.17898
1908	17.0	14.11	167	2.22272
1909	17.9	14.86	176	2.24551
1910	18.9	14.74	175	2.24304
1911	19.8	15.05	178	2.25042
1912	20.8	15.60	185	2.26717
1913	21.7	15.84	188	2.27416
1914 *	22.6	16.27	193	2.28556
1915 *	23.6	17.46	207	2.31597

Source: [a]<u>Abstract of the Census of Manufactures</u>, U. S. Department of Commerce, 1914, p. 16.

* Estimated.

Table 33. Wages and Salaries in Manufacturing Industry: U.S.A.

Year	Dollars[a] (billions)	Value in 1899 Dollars	Index 1899=100	Log of Index
1899	2.30	2.25	100	2.00000
1900	2.50	2.35	104	2.01703
1901	2.71	2.49	111	2.04532
1902	2.92	2.63	117	2.06819
1903	3.12	2.68	119	2.07555
1904	3.33	2.90	129	2.11059
1905	3.54	3.08	137	2.13672
1906	3.75	3.15	140	2.14613
1907	3.96	3.13	139	2.14301
1908	4.16	3.45	153	2.18469
1909	4.37	3.63	161	2.20683
1910	4.58	3.57	159	2.20140
1911	4.78	3.63	161	2.20683
1912	4.98	3.74	166	2.22011
1913	5.19	3.79	168	2.22531
1914 *	5.39	3.88	172	2.23553
1915 *	5.60	4.14	184	2.26483

Source: [a]Abstract of the Census of Manufactures, U. S. Department of Commerce, 1914, p. 16.

* Estimated.

Table 34. Raw Material Consumption in Manufacturing Industry: U.S.A.

Year	Dollars[a] (billions)	Value in 1899 Dollars	Index 1899=100	Log of Index
1899	6.23	6.10	100	2.00000
1900	6.79	6.38	105	2.02119
1901	7.34	6.75	111	2.04532
1902	7.90	7.11	117	2.06819
1903	8.46	7.28	119	2.07555
1904	9.00	7.83	128	2.10721
1905	9.57	8.33	137	2.13672
1906	10.11	8.49	139	2.14301
1907	10.67	8.43	138	2.13988
1908	11.21	9.30	152	2.18184
1909	11.77	9.77	160	2.20412
1910	12.32	9.61	158	2.19866
1911	12.88	9.79	160	2.20412
1912	13.43	10.07	165	2.21748
1913	14.00	10.22	168	2.22531
1914*	14.55	10.48	172	2.23553
1915*	15.10	11.17	183	2.26245

Source: [a]*Abstract of the Census of Manufactures*, U. S. Department of Commerce, 1914, p. 16.

* Estimated.

Table 35. Electrical Energy Consumption in Manufacturing Industry: U.S.A.

Year	Dollars[a] (millions)	Value in 1899 Dollars	Index 1899=100	Log of Index
1899	63.0	61.7	100	2.00000
1900	68.0	63.9	104	2.01703
1901	70.0	64.4	104	2.01703
1902	74.0	66.6	108	2.03342
1903	78.0	67.1	109	2.03743
1904	80.0	69.6	113	2.05308
1905	83.0	72.2	117	2.06819
1906	90.0	75.6	122	2.08636
1907	98.0	77.4	125	2.09691
1908	108.0	89.6	145	2.16137
1909	121.0	100.4	163	2.21219
1910	137.0	106.9	173	2.23805
1911	159.0	120.8	196	2.29226
1912	181.0	135.8	220	2.34242
1913	207.0	151.1	245	2.38917
1914	237.0	170.6	276	2.44091
1915	271.0	200.5	325	2.51188

Source: [a]*Historical Statistics of the United States*, U. S. Department of Commerce, Series G 225-233, p. 159.

Table 36. Time Series
for Model II - U.S.A.

Year	Time	Log of Time	Log of $(\text{Time})^2$
1899	1	0.00000	0.00000
1900	2	0.30103	0.09062
1901	3	0.47712	0.22764
1902	4	0.60206	0.36248
1903	5	0.69897	0.48856
1904	6	0.77815	0.60552
1905	7	0.84510	0.71419
1906	8	0.90309	0.81557
1907	9	0.95424	0.91057
1908	10	1.00000	1.00000
1909	11	1.04139	1.08450
1910	12	1.07918	1.16463
1911	13	1.11394	1.24086
1912	14	1.14613	1.31361
1913	15	1.17609	1.38319
1914	16	1.20412	1.44990
1915	17	1.23045	1.51401

SAMPLE CALCULATIONS

(a) $$\bar{S}^2_{1.2\cdot\cdot n} = \frac{N\sigma_Z{}^2}{N - n}$$

$\bar{S}$ = unbiased standard error of the estimate

N = number of observations

n = total number of variables

$\sigma_Z{}^2$ = variance of residuals (coded) of the actual and estimated values of the dependent variable.

<u>Model I - Mexico</u>

$$\bar{S}^2_{1.2\cdot\cdot n} = \frac{(17)\ (5858.3716)}{17 - 7} = 9959.2317 \text{ (coded)}$$

$$\bar{S}_{1.2\cdot\cdot n} = 99.80 \text{ (coded)}$$

$$\bar{S}_{1.2\cdot\cdot n} = 0.0100 \text{ (uncoded)}$$

(b) $$\bar{R}^2_{1.2\cdot\cdot n} = 1 - \left[\left(\frac{\bar{S}^2_{1.2\cdot\cdot n}}{\sigma_1{}^2} \right) \left(\frac{N - 1}{N} \right) \right]$$

$\bar{R}_{1.2\cdot\cdot n}$ = unbiased multiple correlation coefficient

$\bar{R}^2_{1.2\cdot\cdot n}$ = unbiased coefficient of multiple determination

σ_1 = standard error of the independent variable

Model I - Mexico

$$\bar{R}^2_{1.2\cdots n} = 1.0 - \left[0.01483 \times \frac{16}{17}\right]$$

$$\bar{R}^2_{1.2\cdots n} = 1.0 - (0.01396)$$

$$\bar{R}^2_{1.2\cdots n} = 0.98604$$

$$\bar{R}_{1.2\cdots n} = 0.99301$$

(c) $$t_{1j} = \frac{b_{1j} - \beta_{1j}}{s_{b_{1j}}}$$

b_{1j} = regression coefficient of independent variable j

β_{1j} = regression coefficient of the population

$s_{b_{1j}}$ = standard error of the regression coefficient

Model I - Mexico

$$t_{12} = \frac{b_{12} - \beta_{12}}{s_{b_{12}}} \qquad \text{where } H_o: \ \beta_{12} = 0$$

$$t_{12} = \frac{+0.51456-0}{0.46044} = +1.11753$$

(d) $$F = \frac{\bar{R}^2_{1.2\cdots n}\ (N - n)}{(1 - \bar{R}^2_{1.2\cdots n})\ (n - 1)}$$

$\bar{R}^2_{1.2\cdots n}$ = coefficient of multiple determination

N = number of observations

n = total number of variables

<u>Model I - Mexico</u>

$$F = \frac{(0.98604)(10)}{(1-0.98604)(6)} = \frac{9.86040}{0.08376} = 117.72$$

where N = 17

n = 7

BIBLIOGRAPHY

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(Literature Cited)

1. Douglas, P. H., and Cobb, C. W., "A Theory of Production," The American Economic Review, 18, 1928, p. 139.

2. Heady, E. O., Johnson, G. L., and Hardin, L. S., Resource Productivity, Returns to Scale, and Form Size, 1st ed., Ames, Iowa: The Iowa State College Press, 1956, p. 8.

3. Davidson, R. K., Smith, V. L., and Wiley, J. W., Economics: An Analytical Approach, 1st ed., Homewood, Illinois: Richard D. Irwin, Inc., 1958, p. 328.

4. Heady, Johnson, and Hardin, op. cit., p. 84.

5. German Parra, M., La Industrializacion de Mexico, Mexico D.F.: Imprenta Universitaria, 1954.

6. Schultz, H., The Theory and Measurement of Demand, Chicago, Illinois: The University of Chicago Press, 1958, p. 184.

7. Ibid., p. 193.

8. Tintner, G., Econometrics, New York: John Wiley and Sons, Inc., 1954, p. 303.

9. Heady, Johnson, and Hardin, op. cit., p. 149.

(Other References)

1. Clark, C., The Conditions of Economic Progress, 3rd ed., London: MacMillan and Co., Ltd., 1957.

2. Croxton, E. F., and Cowden, D. J., Applied General Statistics, New York: Prentice Hall, Inc., 1940.

3. Davis, H. T., The Theory of Econometrics, Bloomington, Indiana: The Principia Press, Inc., 1941

4. Douglas, P. H., The Theory of Wages, 1st ed., New York: The MacMillan Company, 1934.

5. Douglas, P. H., and Daly, P., "The Production Function for Canadian Manufacture," The Journal of the American Statistical Association, 39, 1943, p. 178.

6. Douglas, P. H., and Gunn, G., "The Production Function for American Manufacturing in 1919," *The American Economic Review*, *31*, 1941, p. 67.

7. Durand, D., "Some Thoughts on Marginal Productivity with Special Reference to Professor Douglas' Analysis," *Journal of Political Economy*, *45*, 1937, p. 740.

8. Ezekiel, M., *Methods of Correlation Analysis*, 2nd ed., New York: John Wiley and Sons, Inc., 1956.

9. Fabricant, S., *The Output of Manufacturing Industries, 1899-1937*, New York: National Bureau of Economic Research, Inc., 1940.

10. Franco, J., "Productivity and Economic Development in Latin America," *International Labor Review*, *72*, 1955, p. 367.

11. Frisch, R., *Statistical Confluence Analysis by Means of Complete Regression Systems*, Oslo, Norway.

12. Glover, J. G., and Cornell, W. B., *The Development of American Industries*, 3rd ed., New York: Prentice Hall, Inc., 1955.

13. Mendershausen, H., "On the Significance of Professor Douglas' Production Function," *Econometrica*, *6*, 1938, p. 143.

14. Mills, C. F., *Statistical Methods*, New York: Henry Holtand Co., 1924.

15. Mosk, S. A., *Industrial Revolution in Mexico*, Los Angeles, California: University of California Press, 1950.

16. Nacional Financiera, *Informe Anual*, Mexico, D.F., 1957.

17. Phelps-Brown, E. H., "The Meaning of the Fitted Cobb-Douglas Function," *The Quarterly Journal of Economics*, *4*, 1957, p. 263.

18. Richardson, C. H., *An Introduction to Statistical Analysis*, 3rd ed., New York: Harcourt, Brace and Co., 1944.

19. Robinson, J., "Production Function," *The Economic Journal*, *65*, 1955, p. 263.

20. Tilles, S., "Productivity in Underdeveloped Countries," *International Labor Review*, *72*, 1955, p. 496.